Tutorial 1

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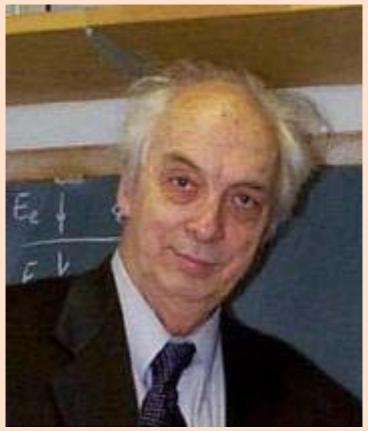
Nano-Optics and Plasmonics:

Promises and Challenges

Basics

- Metal Dielectric and Metamaterials
- Response function theory and Kramers-Kronig relations
- Parallel between Optics and Quantum Mechanics
- Wigner Delay, GH shift and Hartman effect
- Stratified Media, Surface and Guided modes
- Optical invisibility cloaks
- Metasurfaces: recent trends

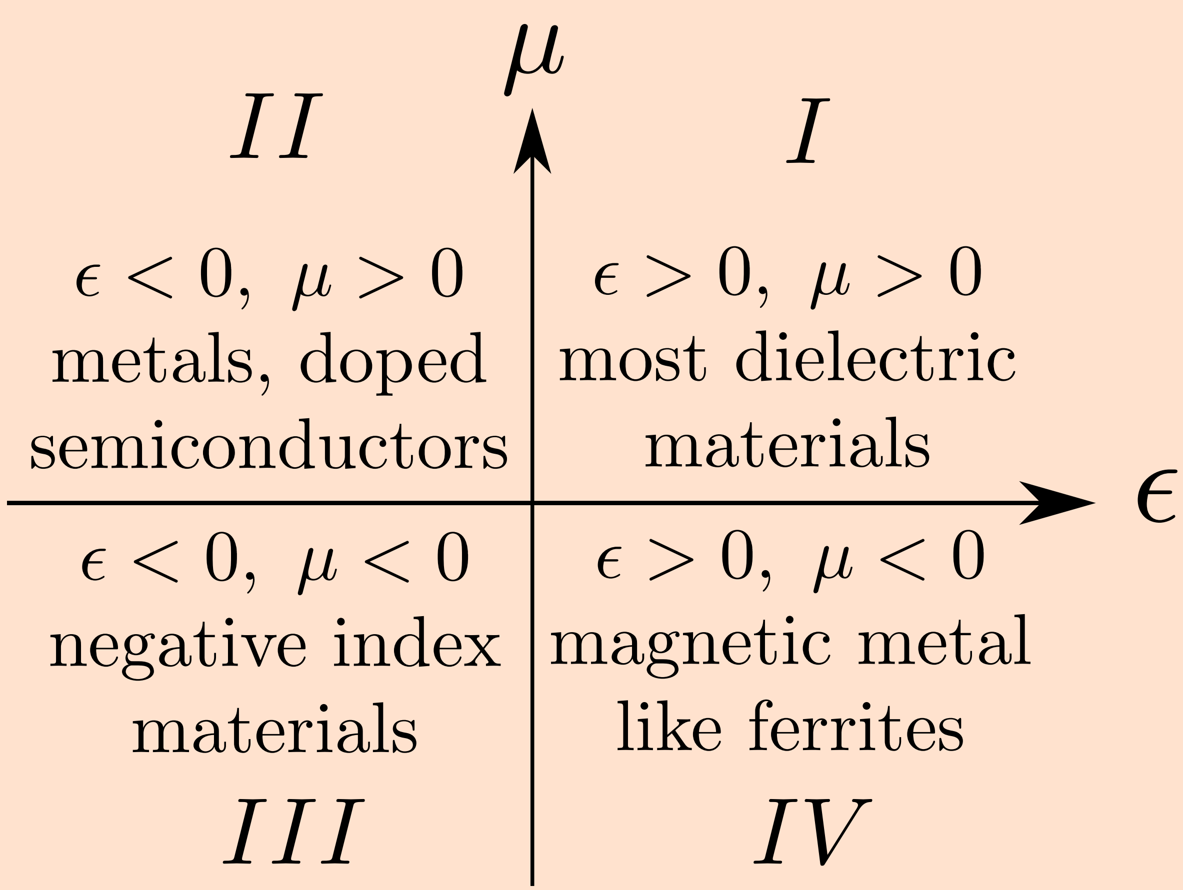
Metamaterials and NIM



Victor Veselago 1968

$$\epsilon < 0 \quad \text{Permittivity}$$

$$\mu < 0 \quad \text{Permeability}$$



Sir John Penury 2000

Lorentz and Drude models

$$P = Np = -Nex.$$

$$F = -eE(z,t) = -eE_0 \cos(kz - \omega t).$$

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = -\frac{eE_0}{m} e^{-i\omega t}.$$

$$x(t) = \frac{-e/m}{[(\omega_0^2 - \omega^2) - (2i\gamma\omega)]} E_0 e^{-i\omega t} = \frac{-e/m}{[(\omega_0^2 - \omega^2) - (2i\gamma\omega)]} E(t)$$

$$P(t) = \frac{Ne^2/m}{[(\omega_0^2 - \omega^2) - (2i\gamma\omega)]} E(t).$$

$$P(t) = \varepsilon_0 \chi E(t).$$

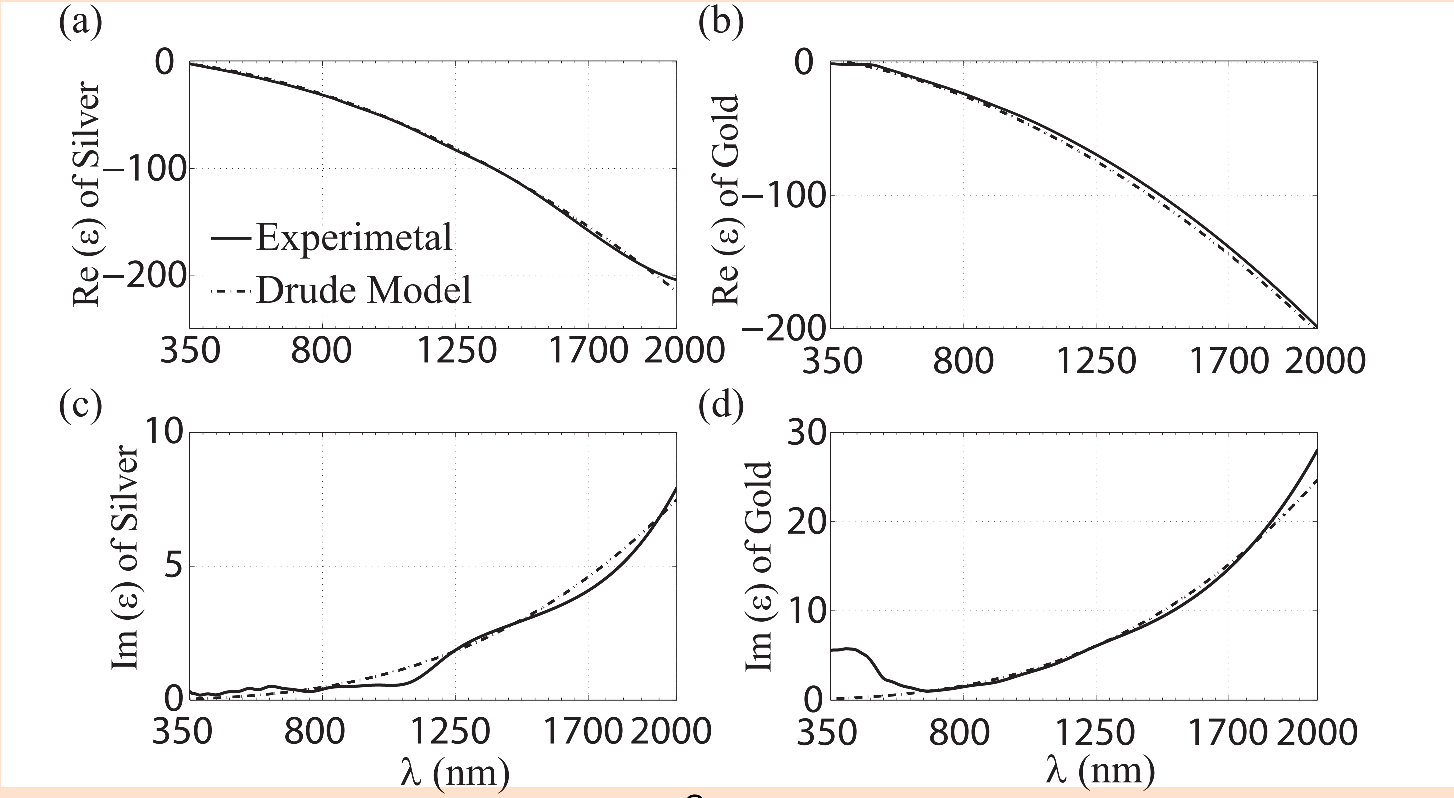
$$\chi(\omega) = \frac{Ne^2/(m\varepsilon_0)}{[(\omega_0^2 - \omega^2) - (2i\gamma\omega)]}.$$

Drude model

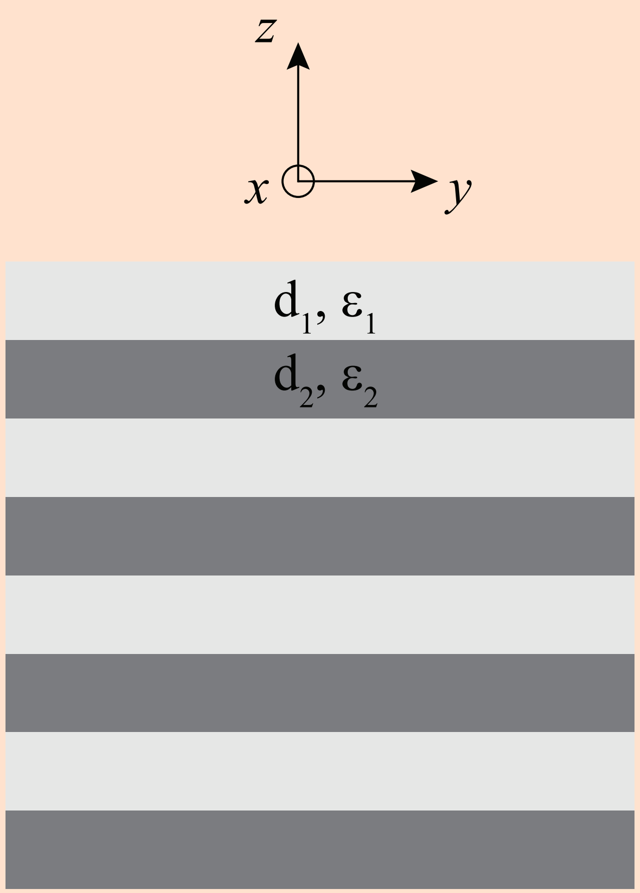
$$\chi_e = -\frac{\omega_p^2}{\omega(\omega + 2i\gamma)},$$

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + 2i\gamma)},$$

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + (2\gamma)^2} + i\frac{2\gamma\omega_p^2}{\omega(\omega^2 + (2\gamma)^2)}.$$



Motivation for plasmonic structures
Composite media



$$\varepsilon_{||} = f_1\varepsilon_1 + f_2\varepsilon_2,$$

$$\frac{1}{\varepsilon_{\perp}} = \frac{f_1}{\varepsilon_1} + \frac{f_2}{\varepsilon_2}.$$

Highly anisotropic medium
Hyperbolic medium

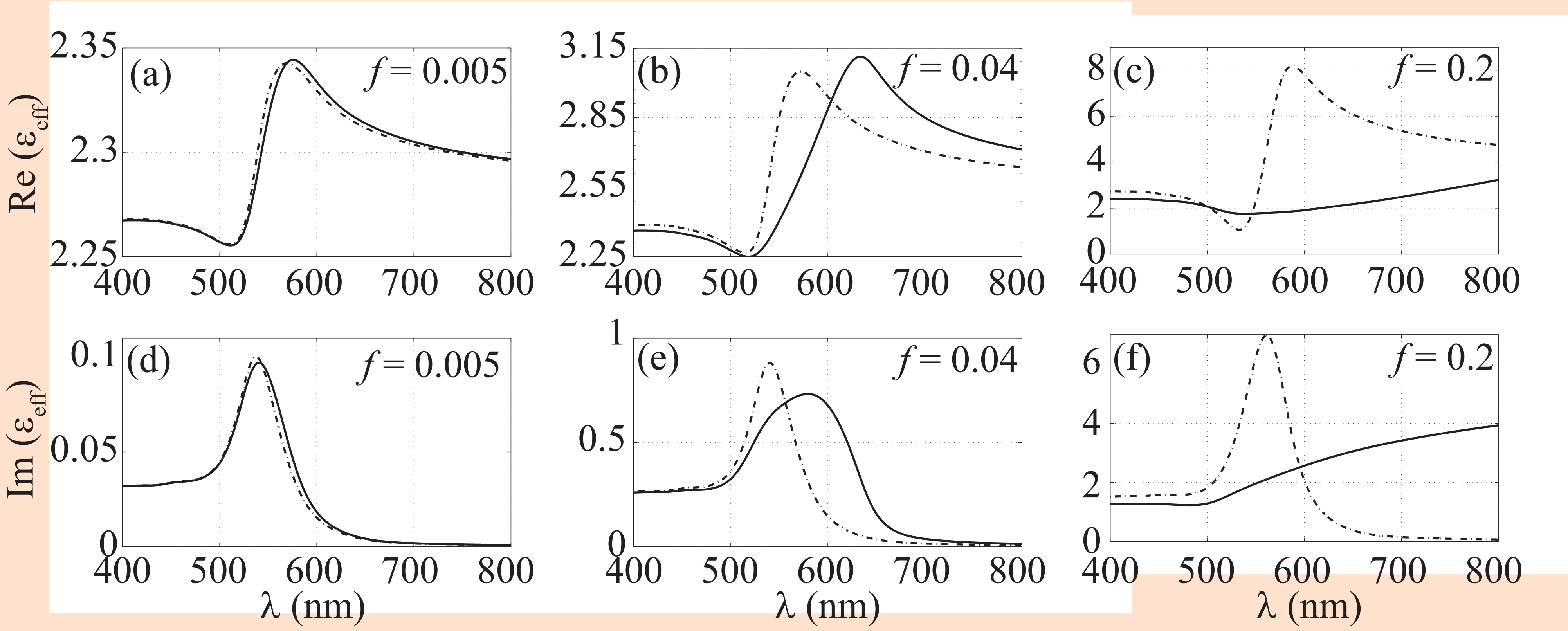
Granular multi-component composite media

Maxwell-Garnett
or
Bruggemann composites

$$\epsilon_{eff} = \epsilon_h + 3f\epsilon_h \frac{\epsilon_1 - \epsilon_h}{\epsilon_1 + 2\epsilon_h}$$

$$\epsilon_1 + 2\epsilon_h = 0$$

Localized plasmon resonances



Linear Response Function Theory

Time domain

$$\mathbf{P}(t) = \epsilon_0 \int_0^\infty \bar{\bar{\mathbf{R}}}(\tau) \cdot \mathbf{E}(t-\tau) d\tau,$$

$$\bar{\bar{\mathbf{R}}}(\tau) = 0, \quad \text{for} \quad \tau < 0.$$ Causality

$$P_i(t) = \epsilon_0 \sum_j \int R_{ij}(\tau) E_j(t-\tau)\ d\tau.$$

Frequency domain

$$\boldsymbol{\mathcal{E}}(\omega) = \int_{-\infty}^{\infty} \mathbf{E}(t) e^{i\omega t}\ dt,$$

$$\mathbf{E}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \boldsymbol{\mathcal{E}}(\omega) e^{-i\omega t}\ d\omega.$$

Linear Response Function Theory contd.

$$\mathbf{P}(t) = \epsilon_0 \int_0^\infty d\tau \; \bar{\bar{\mathbf{R}}}(\tau) \cdot \left[\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \; \mathbf{E}(\omega) e^{-i\omega(t-\tau)} \right],$$

$$= \epsilon_0 \int_{-\infty}^{\infty} \left\{ \int_0^\infty \bar{\bar{\mathbf{R}}}(\tau) e^{i\omega\tau} d\tau \right\} \cdot \mathbf{E}(\omega) \; e^{-i\omega t} \; \frac{d\omega}{2\pi}.$$

We introduce the linear electric susceptibility $\bar{\bar{\boldsymbol{\chi}}}_e(\omega)$ as

$$\bar{\bar{\boldsymbol{\chi}}}_e(\omega) = \int_0^\infty \bar{\bar{\mathbf{R}}}(\tau) e^{i\omega\tau} d\tau,$$

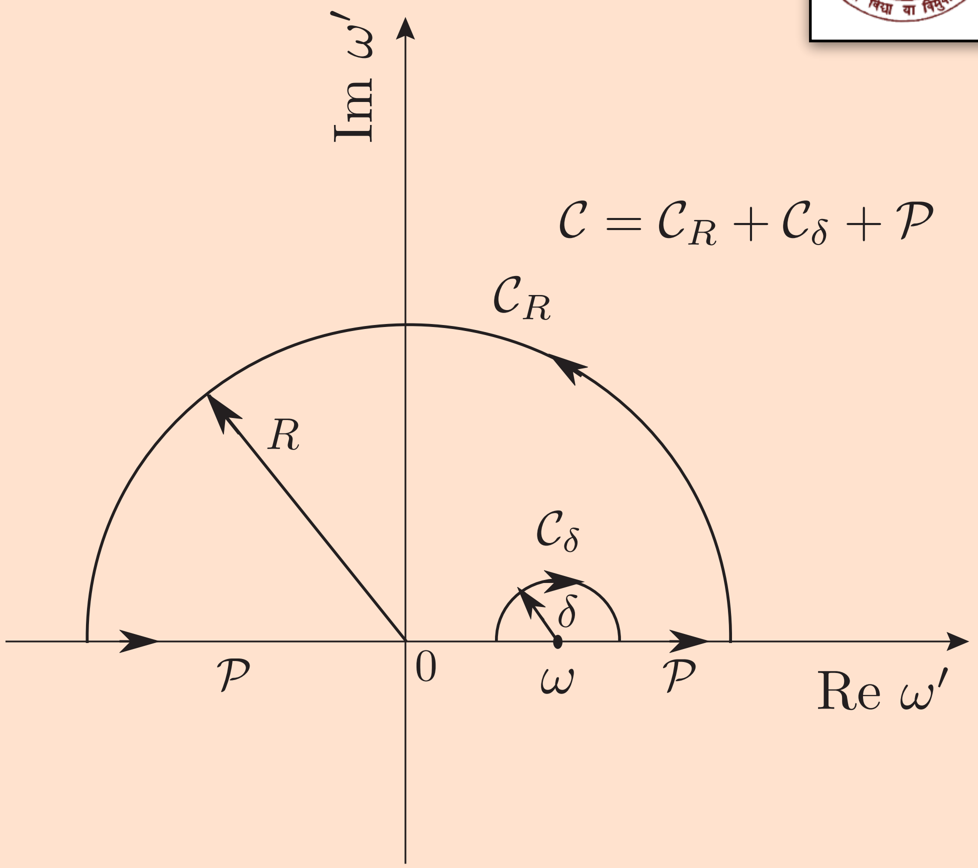
$$\mathbf{P}(t) = \epsilon_0 \int_{-\infty}^{\infty} \bar{\bar{\boldsymbol{\chi}}}_e(\omega) \cdot \mathbf{E}(\omega) \; e^{-i\omega t} \; \frac{d\omega}{2\pi}.$$

Causality in frequency domain:
susceptibility analytic in upper half complex plane

Kramers-Kronig relations contd

$$\int_{\mathcal{C}} \frac{\chi(\omega')}{\omega' - \omega} d\omega',$$

$$0 = \mathrm{P} \int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega' - \omega} d\omega' - i\pi\chi(\omega).$$



$$\chi(\omega) = \chi' + i\chi'' = \frac{-i}{\pi} \mathrm{P} \int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega' - \omega} d\omega',$$

$$\chi'(\omega) = \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{\infty} \frac{\chi''(\omega')}{\omega' - \omega} d\omega',$$

$$\chi''(\omega) = \frac{-1}{\pi} \mathrm{P} \int_{-\infty}^{\infty} \frac{\chi'(\omega')}{\omega' - \omega} d\omega'.$$

Kramers-Kronig relations contd: symmetry for real frequencies

$$\chi^*(\omega) = \int_0^\infty R(\tau) e^{-i\omega^*\tau} d\tau.$$

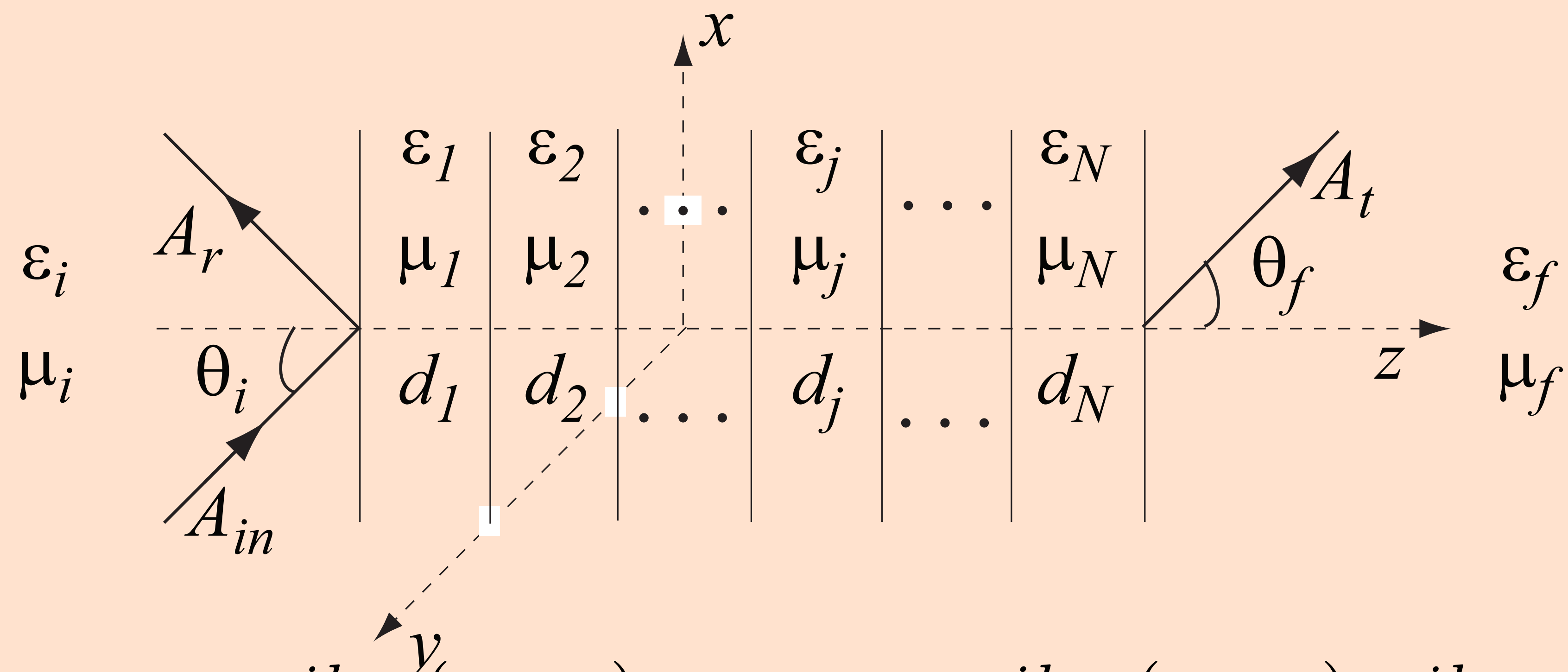
$$\chi^*(\omega) = \chi(-\omega^*).$$

$$\chi^*(\omega) = \chi(-\omega),$$

$$\chi'(\omega) = \chi'(-\omega), \quad \chi''(\omega) = -\chi''(-\omega).$$

$$\begin{aligned}
\chi'(\omega) &= \frac{1}{\pi}\,\mathrm{P}\int_{-\infty}^{\infty} \frac{\chi''(\omega')(\omega'+\omega)}{\omega'^2-\omega^2} d\omega', \\
&= \frac{1}{\pi}\,\mathrm{P}\left[\int_{-\infty}^{\infty} \frac{\omega'\ \chi''(\omega')}{\omega'^2-\omega^2} d\omega' + \int_{-\infty}^{\infty} \frac{\omega\ \chi''(\omega')}{\omega'^2-\omega^2} d\omega'\right], \\
&= \frac{2}{\pi}\,\mathrm{P}\int_0^{\infty} \frac{\omega'\ \chi''(\omega')}{\omega'^2-\omega^2} d\omega'.
\end{aligned}$$

Stratified medium: reflection and transmission



$$H_{jy} = [A_{j+}e^{ik_{jz}(z-z_j)} + A_{j-}e^{-ik_{jz}(z-z_j)}]e^{ik_x x}e^{-i\omega t},$$

$$\sqrt{\varepsilon_0}E_{jx} = [p_{jz}\sqrt{\mu_0}(A_{j+}e^{ik_{jz}(z-z_j)} - A_{j-}e^{-ik_{jz}(z-z_j)})]e^{ik_x x}e^{-i\omega t}$$

$$k_{jz} = \sqrt{k_0^2 n_j^2 - k_x^2}, \quad k_x = k_0 n_i \sin\theta_i, \quad p_{jz} = \frac{k_{jz}}{k_0 \varepsilon_{rj}},$$

$$\begin{pmatrix} \sqrt{\mu_0}H_{jy} \\ \sqrt{\epsilon_0}E_{jx} \end{pmatrix} = \begin{pmatrix} e^{ik_{jz}z} & e^{-ik_{jz}z} \\ p_{jz}e^{ik_{jz}z} & -p_{jz}e^{-ik_{jz}z} \end{pmatrix} \begin{pmatrix} \sqrt{\mu_0}A_{j+} \\ \sqrt{\mu_0}A_{j-} \end{pmatrix},$$

Stratified medium contd.

$$\begin{pmatrix} H_y \\ E_x \end{pmatrix}_j = M_j \begin{pmatrix} H_y \\ E_x \end{pmatrix}_{j+1}$$

$$\bar{E}_{jx} = \sqrt{\epsilon_0} E_{jx}, \qquad \bar{H}_{jy} = \sqrt{\mu_0} H_{jy}.$$

$$M_j = \begin{pmatrix} \cos(k_{jz} d_j) & -(i/p_{jz}) \sin(k_{jz} d_j) \\ -i p_{jz} \sin(k_{jz} d_j) & \cos(k_{jz} d_j) \end{pmatrix}.$$

$$M_{total} = M_1 M_2 \ldots . M_N.$$

$$\begin{pmatrix} H_y \\ E_x \end{pmatrix}_{z=0} = M_{total} \begin{pmatrix} H_y \\ E_x \end{pmatrix}_{z=d_N}.$$

Stratified medium contd.

$$\begin{pmatrix} H_y \\ E_x \end{pmatrix}_{z=0} = \begin{pmatrix} 1 & 1 \\ p_{iz} & -p_{iz} \end{pmatrix} \begin{pmatrix} \sqrt{\mu_0} A_{in} \\ \sqrt{\mu_0} A_r \end{pmatrix},$$

$$\begin{pmatrix} H_y \\ E_x \end{pmatrix}_{z=d_N} = \begin{pmatrix} 1 \\ p_{fz} \end{pmatrix} \sqrt{\mu_0} A_t,$$

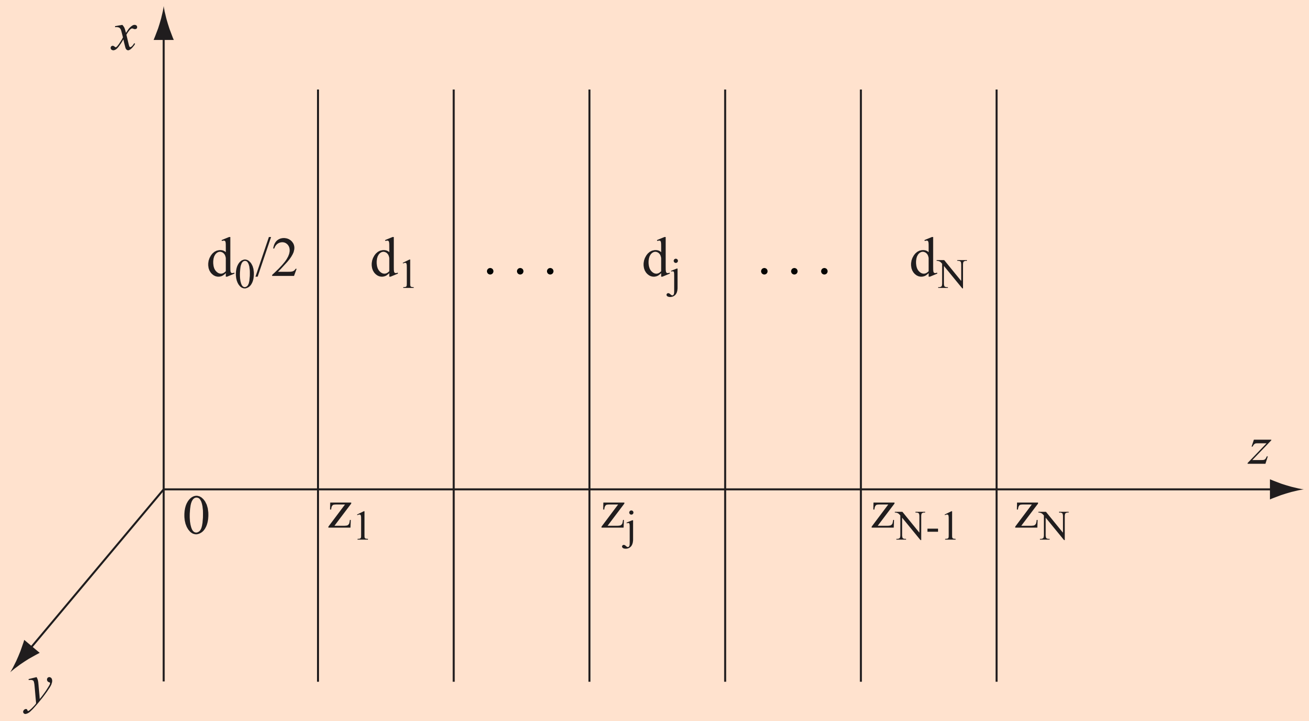
$$\begin{pmatrix} 1 & 1 \\ p_{zi} & -p_{zi} \end{pmatrix} \begin{pmatrix} A_{in} \\ A_r \end{pmatrix} = M_{total} \begin{pmatrix} 1 \\ p_{zf} \end{pmatrix} A_t.$$

$$r = \frac{A_r}{A_{in}} = \frac{(m_{11} + m_{12}p_f)p_i - (m_{21} + m_{22}p_f)}{(m_{11} + m_{12}p_f)p_i + (m_{21} + m_{22}p_f)},$$

$$t = \frac{A_t}{A_{in}} = \frac{2p_i}{(m_{11} + m_{12}p_f)p_i + (m_{21} + m_{22}p_f)},$$

$$R = |r|^2, \quad \text{and} \quad T = \frac{p_f}{p_i} |t|^2,$$

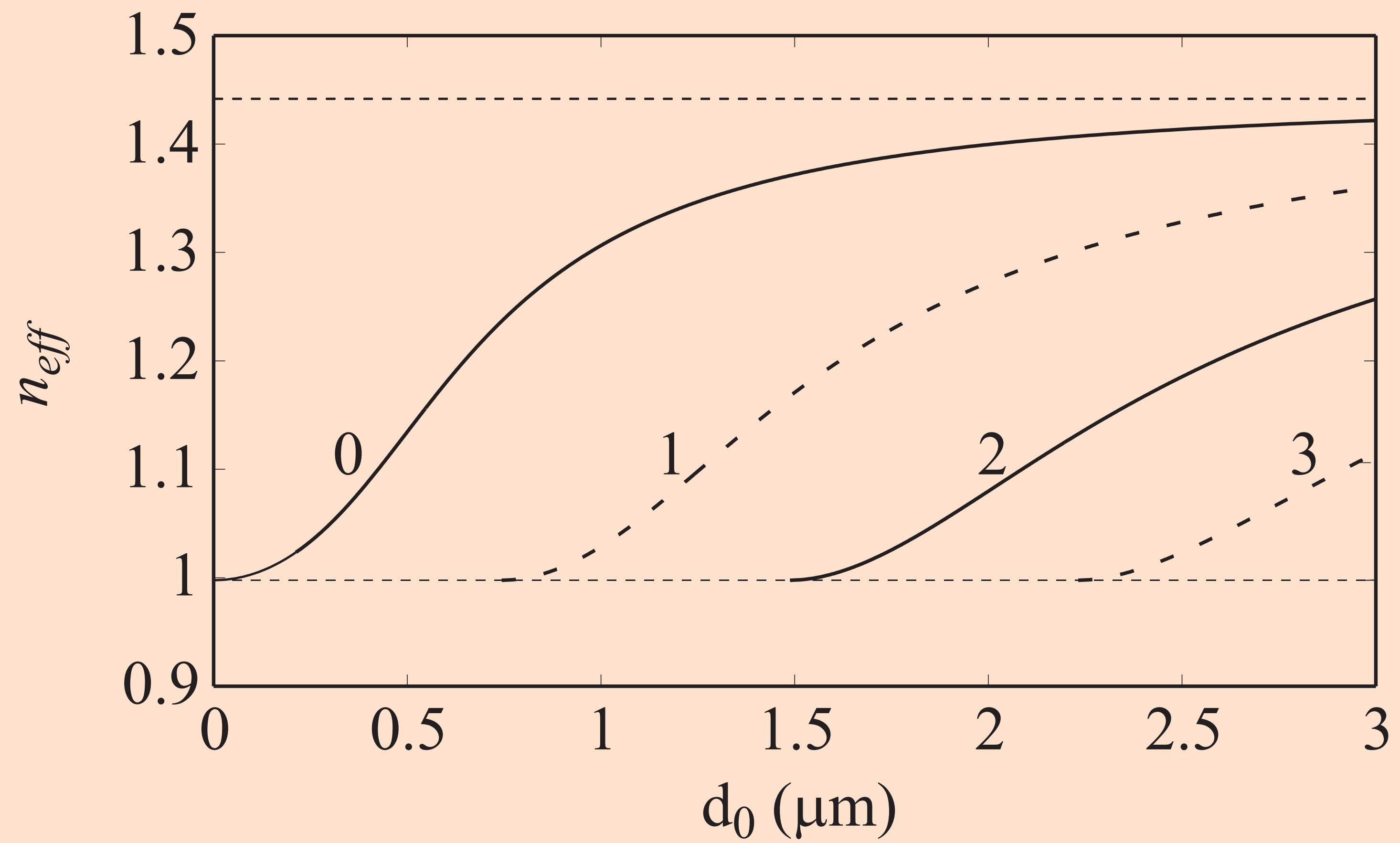
Symmetric stratified medium



$$
\begin{aligned}
H_{0y} &= A_0(e^{ik_{0z}z} \pm e^{-ik_{0z}z}),\\
E_{0x} &= p_{0z}A_0(e^{ik_{0z}z} \mp e^{-ik_{0z}z}),
\end{aligned}
$$

$$
\begin{pmatrix} 1 & \pm 1 \\ p_{0z} & \mp p_{0z} \end{pmatrix} \begin{pmatrix} A_0 \\ A_0 \end{pmatrix} = M_T \begin{pmatrix} 1 \\ p_{tz} \end{pmatrix} A_t .
$$

Guided modes



$$k_{tz} = i\sqrt{k_x^2 - k_0^2\varepsilon_t} = i\bar{k}_{tz}.$$

$$\begin{aligned} \varepsilon_d \bar{k}_{tz} - \varepsilon_t k_{0z} \tan(k_{0z} d_0/2) &= 0, \\ \varepsilon_d \bar{k}_{tz} + \varepsilon_t k_{0z} \cot(k_{0z} d_0/2) &= 0. \end{aligned}$$

Coupled surface plasmons

$$M_T = M_0(d_0/2)M_1(d_1)\cdots M_j(d_j)\cdots M_N(d_N).$$

for the symmetric mode

$$\begin{aligned} m_{21} + m_{22}p_{tz} &= 0, \\ (m_{11} + m_{12}p_{tz})A_t &= 2A_0, \end{aligned}$$

for the antisymmetric mode

$$\begin{aligned} m_{11} + m_{12}p_{tz} &= 0, \\ (m_{21} + m_{22}p_{tz})A_t &= 2p_{0z}A_0. \end{aligned}$$

Coupled surface plasmons contd.

$$M_T = M_0(d_0/2)M_1(d_1)\cdots M_j(d_j)\cdots M_N(d_N).$$

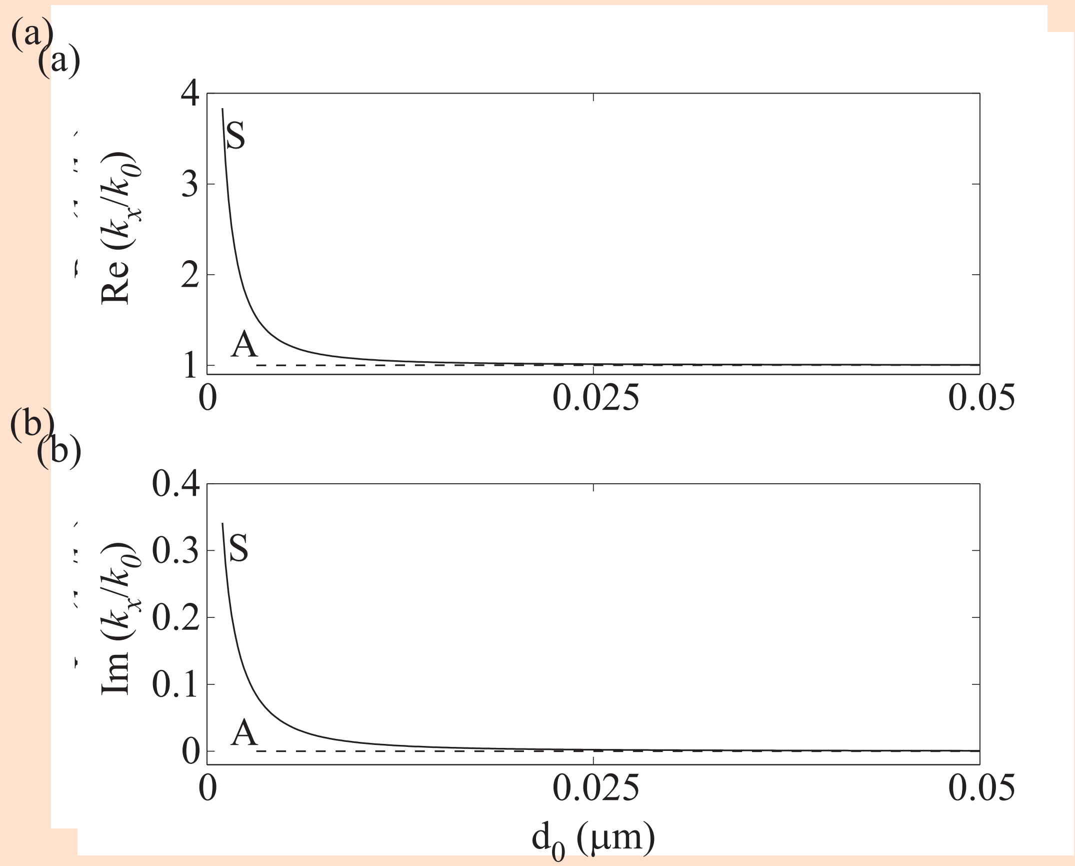
for the symmetric mode

$$\begin{aligned} m_{21} + m_{22}p_{tz} &= 0, \\ (m_{11} + m_{12}p_{tz})A_t &= 2A_0, \end{aligned}$$

for the antisymmetric mode

$$\begin{aligned} m_{11} + m_{12}p_{tz} &= 0, \\ (m_{21} + m_{22}p_{tz})A_t &= 2p_{0z}A_0. \end{aligned}$$

Coupled surface plasmons contd.

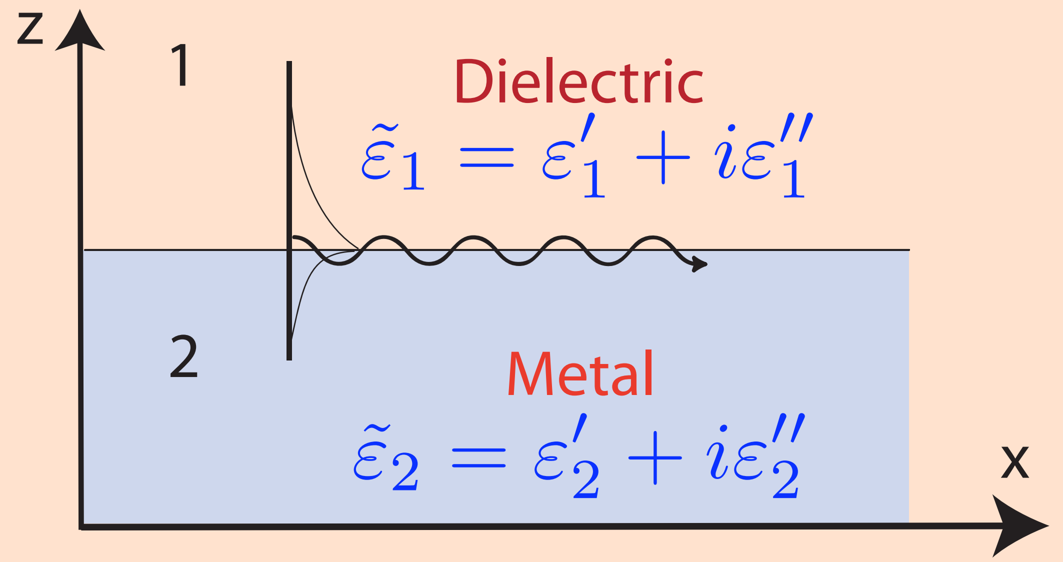


$$k_{0z} = i\sqrt{k_x^2 - k_0^2\varepsilon_m} = i\bar{k}_{0z}, \quad k_{tz} = i\sqrt{k_x^2 - k_0^2\varepsilon_t} = i\bar{k}_{tz}.$$

$$\begin{aligned} \varepsilon_m k_{tz} + \varepsilon_t k_{0z} \tanh(x) &= 0, \\ \varepsilon_m k_{tz} + \varepsilon_t k_{0z} \coth(x) &= 0, \end{aligned}$$

$$x = \frac{\bar{k}_{0z} d_0}{2} = \frac{k_{0z} d_0}{2i}$$

Surface plasmons



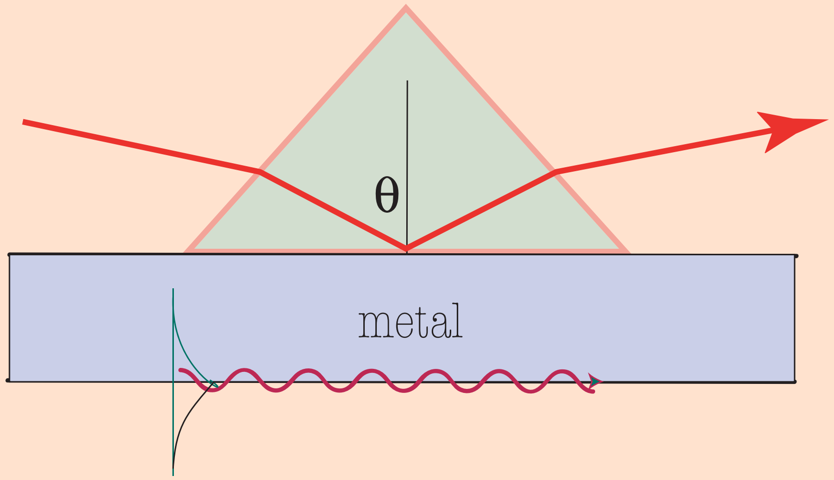
$$\boldsymbol{H} = (0, H_y, 0) \text{ TM p-polarization}$$

$$\boldsymbol{E} = (0, E_y, 0) \text{ TE s-polarization}$$

$$\tilde{k}_x = \tilde{k}_x' + i\tilde{k}_x'' = \frac{\omega}{c_0}\sqrt{\frac{\tilde{\varepsilon}_1\tilde{\varepsilon}_2}{\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2}}$$

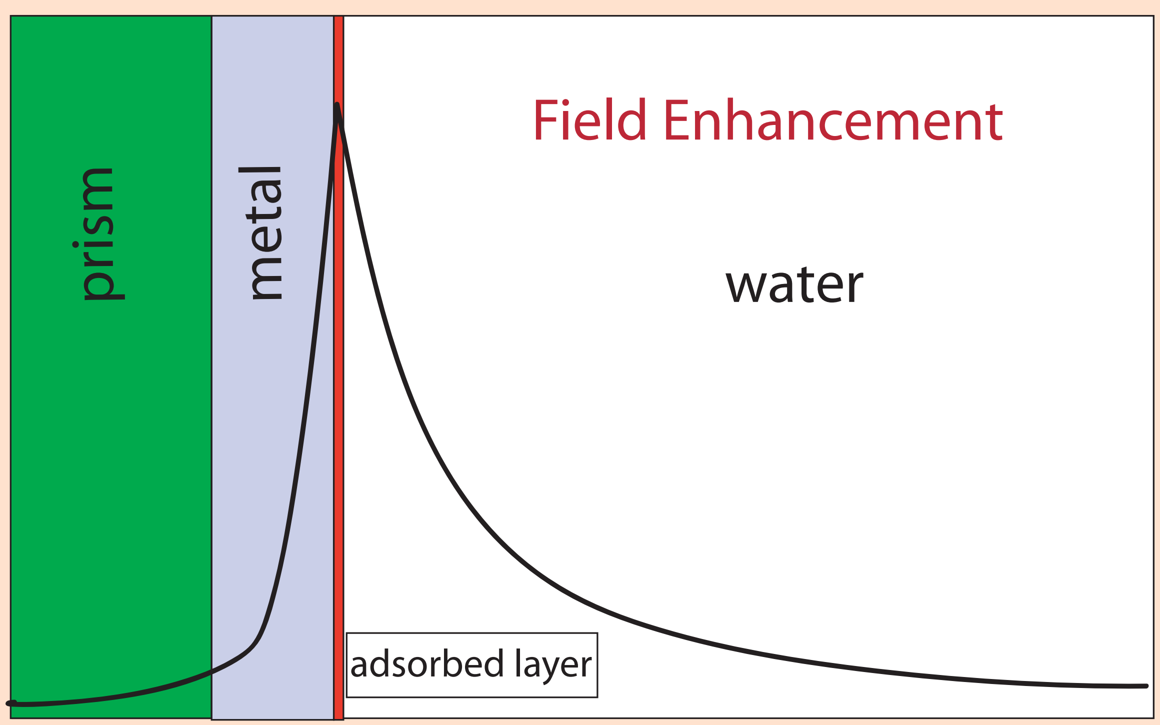
How to excite these modes?

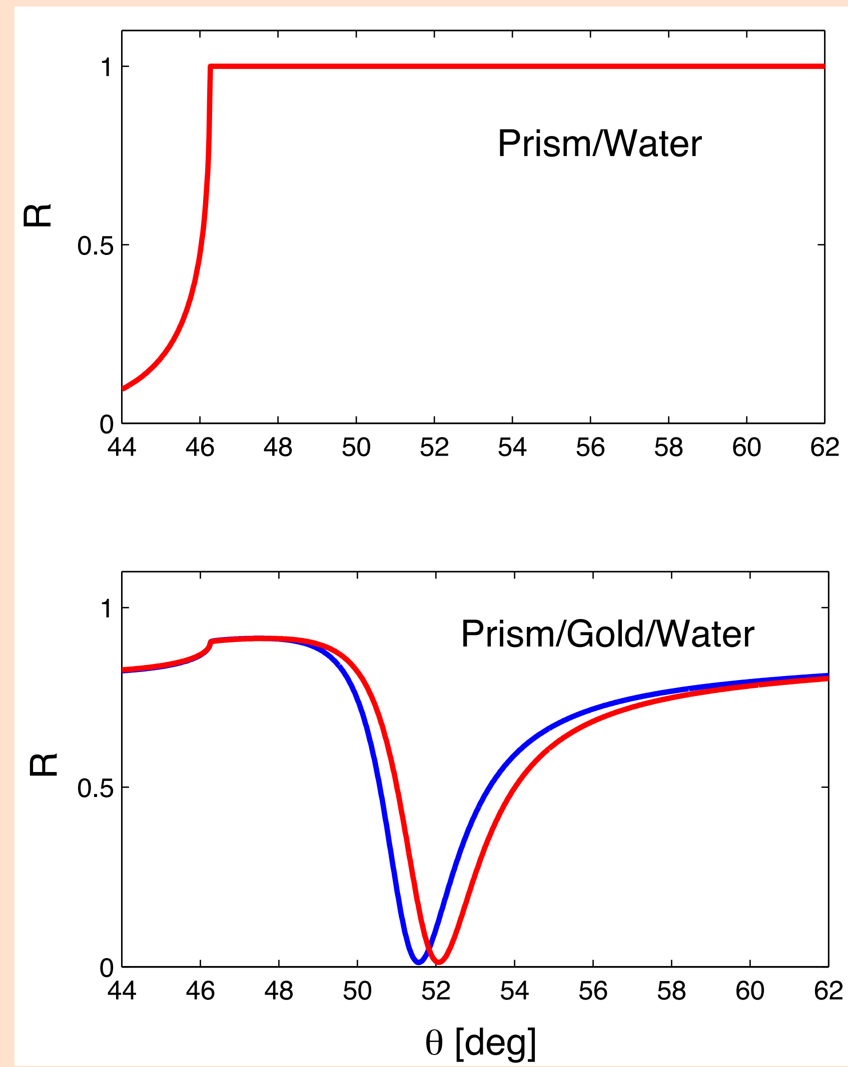
$$k_{in} = k_0 \sqrt{\varepsilon_p} \sin\theta$$



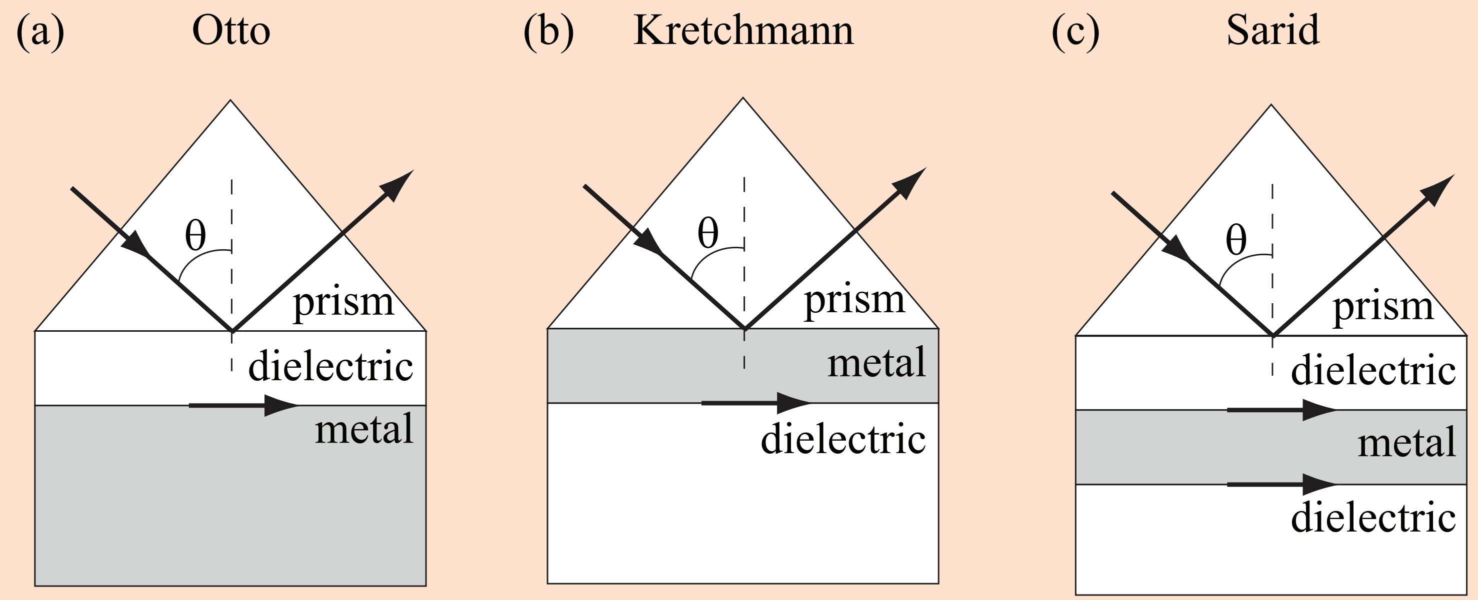
free space

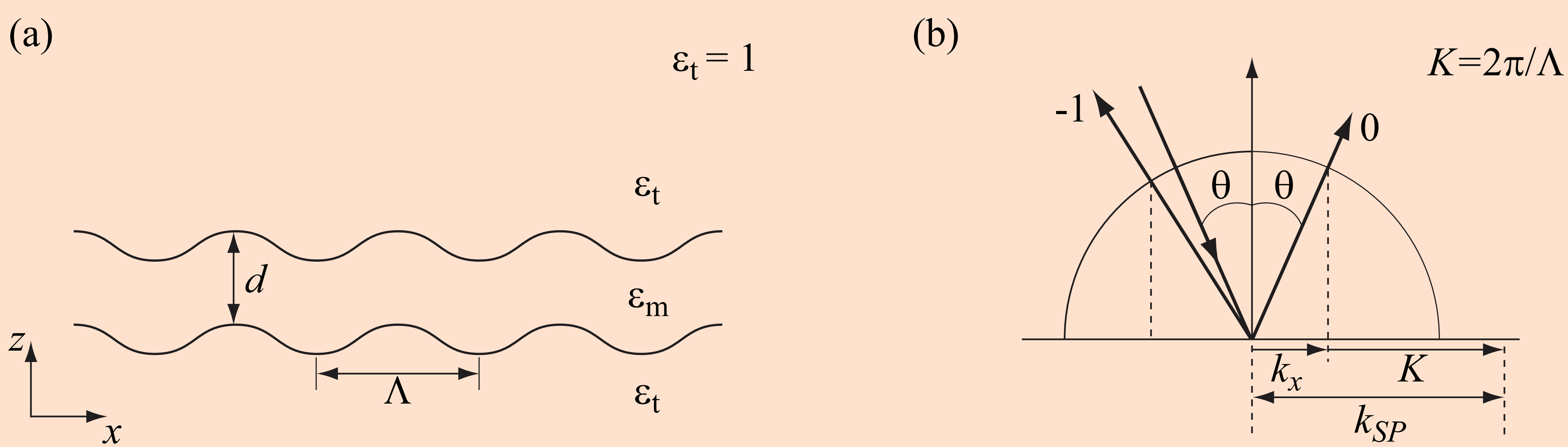
$$k_{in} = k_0 \sin\theta$$





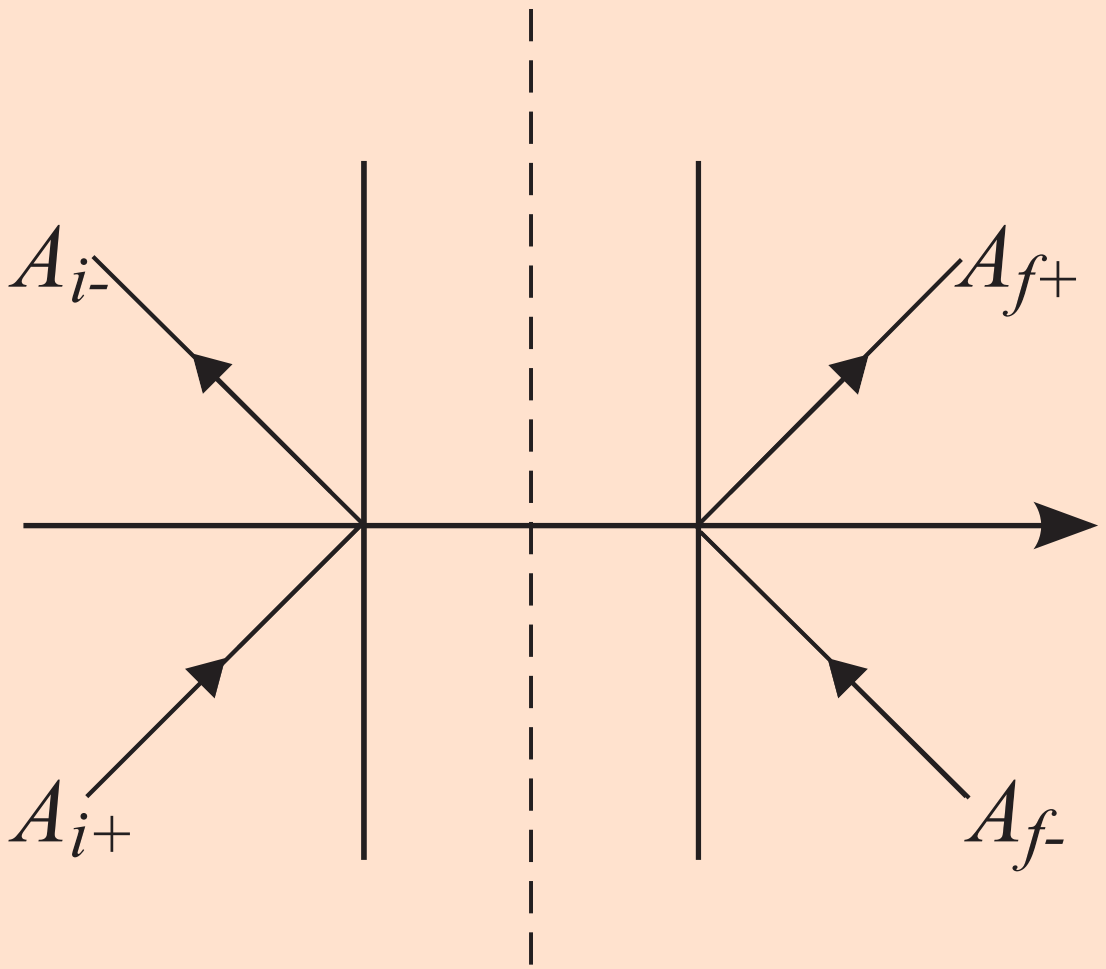
Excitation of guided modes





$$k_{g/sp} = k_0 \sin(\theta) + mK, \quad m = \pm 1, \pm 2, \ldots$$

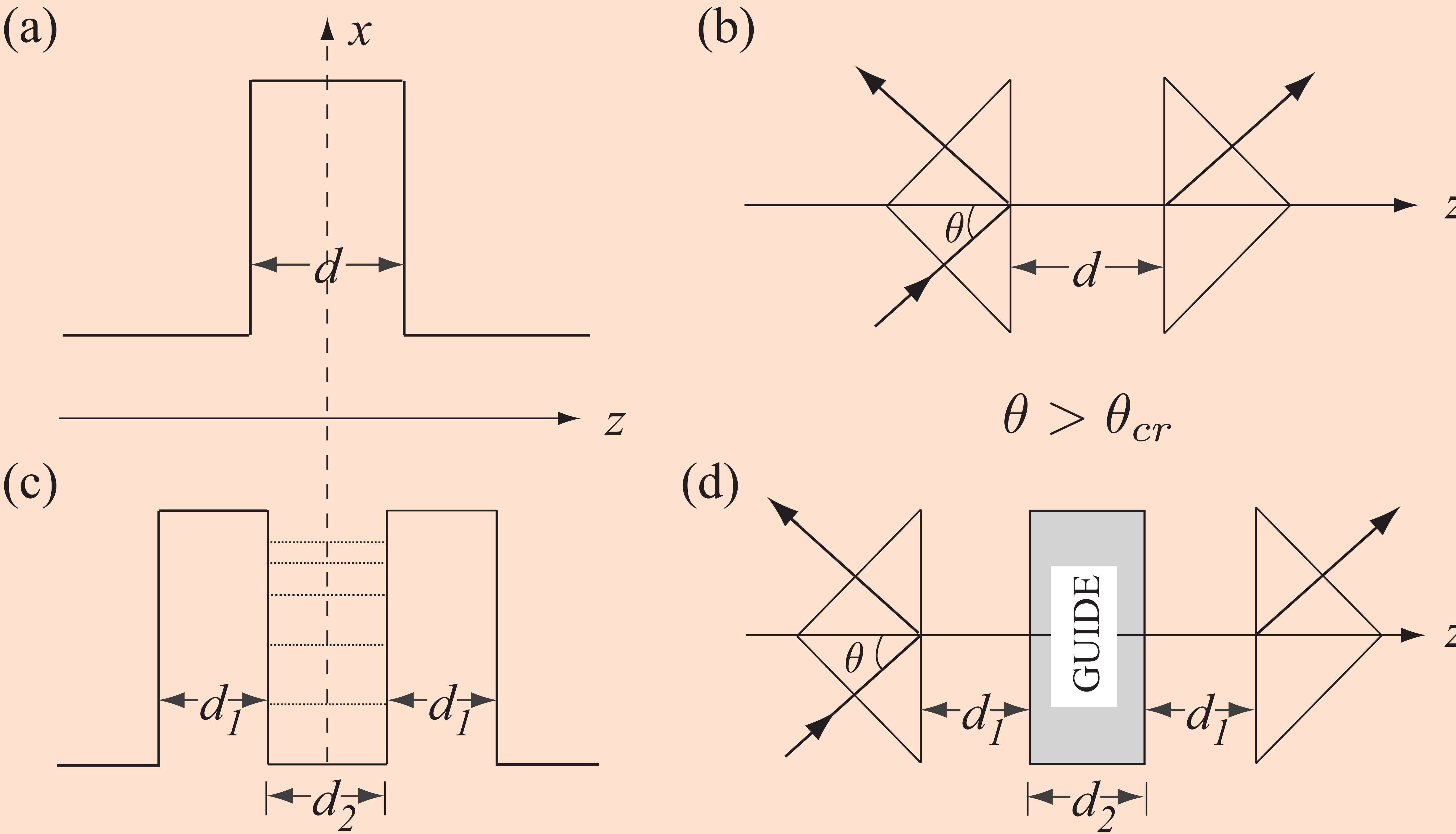
Coherent perfect absorption



$$\begin{pmatrix} 1 & \pm 1 \\ p_{0z} & \mp p_{0z} \end{pmatrix} \begin{pmatrix} A_0 \\ A_0 \end{pmatrix} = M_{d_0/2} \begin{pmatrix} 1 \\ -p_{tz} \end{pmatrix} A_{in},$$

$$\begin{aligned} p_{tz} + i p_{0z} \tan(k_{0z} d_0/2) &= 0, \\ p_{tz} - i p_{0z} \cot(k_{0z} d_0/2) &= 0, \end{aligned}$$

Analogy between electron and photon tunneling



Wigner delay

$$E_I(z,t) = F(t)e^{i(\beta z - \omega_c t)},, \quad F(t) = \int A(\omega)e^{-i(\omega-\omega_c)t}d\omega.$$

$$\tilde{t}(\omega) = |\tilde{t}(\omega)|e^{i\phi_T(\omega)}.$$

$$E_T(z,t) = e^{i(\beta z - \omega_c t)} \int A(\omega)|\tilde{t}(\omega)|e^{i\phi_T(\omega)}e^{-i(\omega-\omega_c)t}d\omega.$$

$$\phi_t(\omega) = \phi_T(\omega_c) + \left.\frac{\partial \phi_T}{\partial \omega}\right|_{\omega_c}(\omega - \omega_c) + \cdots\cdots.$$

$$E_T(z,t) = e^{i(\beta z - \omega_c t + \phi_T(\omega_c))} \int A(\omega)|\tilde{t}(\omega)|e^{-i\left(t - \left.\frac{\partial \phi_T}{\partial \omega}\right|_{\omega_c}\right)(\omega-\omega_c)}d\omega.$$

$$E_T(z,t) = |\tilde{t}(\omega_c)|F\left(t - \left.\frac{\partial \phi_T}{\partial \omega}\right|_{\omega_c}\right)\exp i(\beta z - \omega_c t + \phi_T(\omega_c)).$$

$$\tau_t = \left.\frac{\partial \phi_T}{\partial \omega}\right|_{\omega_c}.$$

Goos-Hanchen shift

$$E_I(x,t) = e^{i(\alpha_0 x + \beta z - \omega_0 t)} \int A(\alpha) e^{i(\alpha-\alpha_0)x} d\alpha.$$

$$E_R(x,t) = e^{i(\alpha_0 x - \beta z - \omega t)} \int A\left(\alpha\right) e^{i(\alpha-\alpha_0)x + i\phi_R(\alpha)} d\alpha,$$

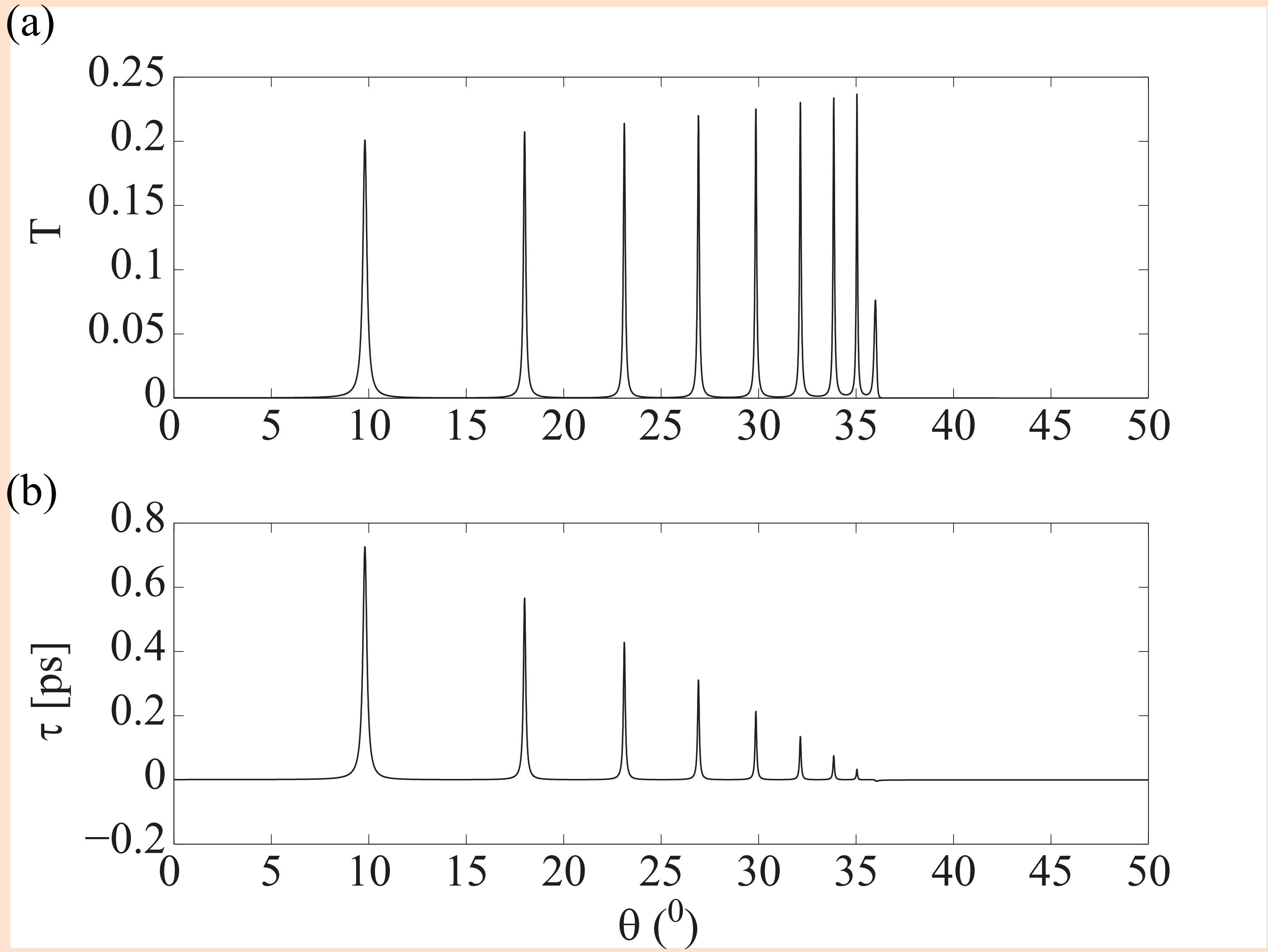
$$E_R(x,t) = e^{i(\alpha_0 x - \beta z - \omega t + \phi_R(\alpha_0))} \int A\left(\alpha\right) e^{i(\alpha-\alpha_0)\left(x + \left.\frac{\partial \phi_R}{\partial \alpha}\right|_{\alpha_0}\right)} d\alpha.$$

$$F(x) = \int A(\alpha) e^{i(\alpha-\alpha_0)x} d\alpha,$$

$$E_R = F\left(x + \left.\frac{\partial \phi_R}{\partial \alpha}\right|_{\alpha_0}\right) e^{i(\alpha_0 x - \beta z - \omega t) + i\phi_R(\alpha_0)},$$

$$d_{GH} = -\left.\frac{\partial \phi_R}{\partial \alpha}\right|_{\alpha_0.} .$$

Resonant tunneling and fast and slow light



Reflectionless potentials

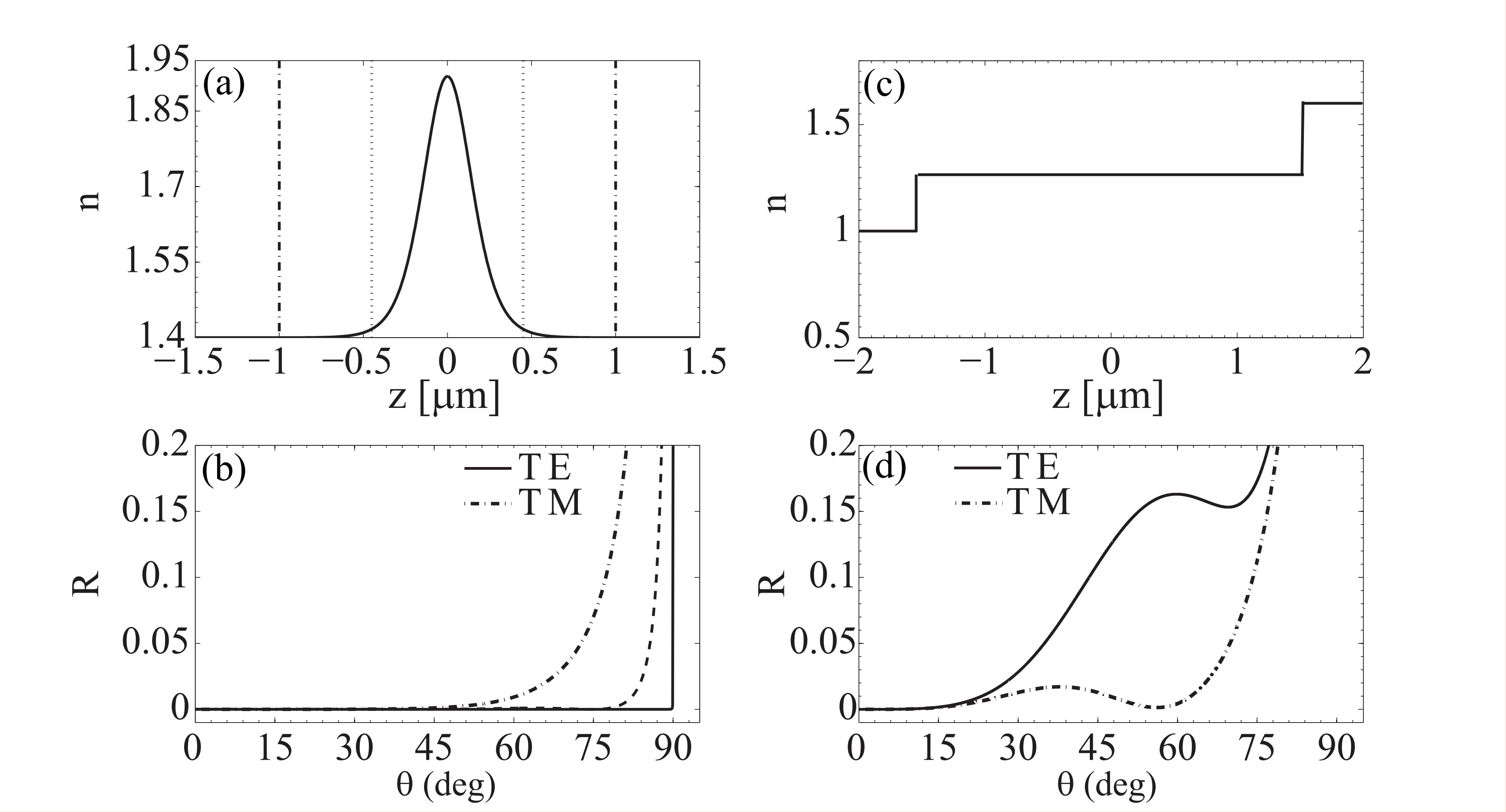
$$\Psi(z,E) \sim e^{i\sqrt{E}z} \quad \text{as} \quad z \to -\infty,$$
$$\Psi(z,E) \sim t e^{i\sqrt{E}z} \quad \text{as} \quad z \to +\infty,$$

$$V(z) = -2\kappa_1^2 \text{sech}(\kappa_1 z).$$

$$\Psi(z) = \frac{i\sqrt{E} - \kappa_1 \tanh(\kappa_1 z)}{i\sqrt{E} + \kappa_1} e^{i\sqrt{E}z},$$

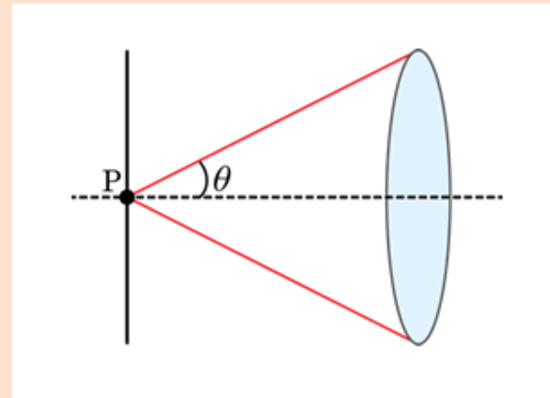
$$\epsilon(z) = n^2(z) = n_s^2 - \frac{V(z)}{k_0}, \quad \epsilon_s = n_s^2.$$

Reflectionless potentials

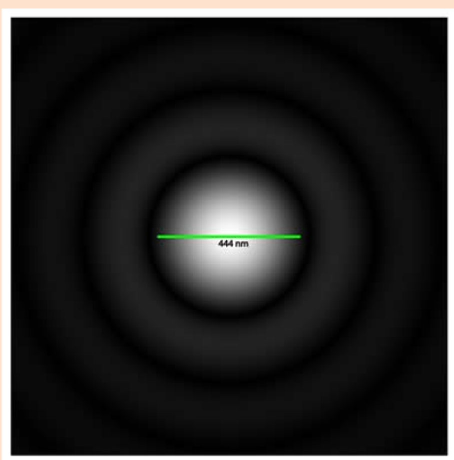


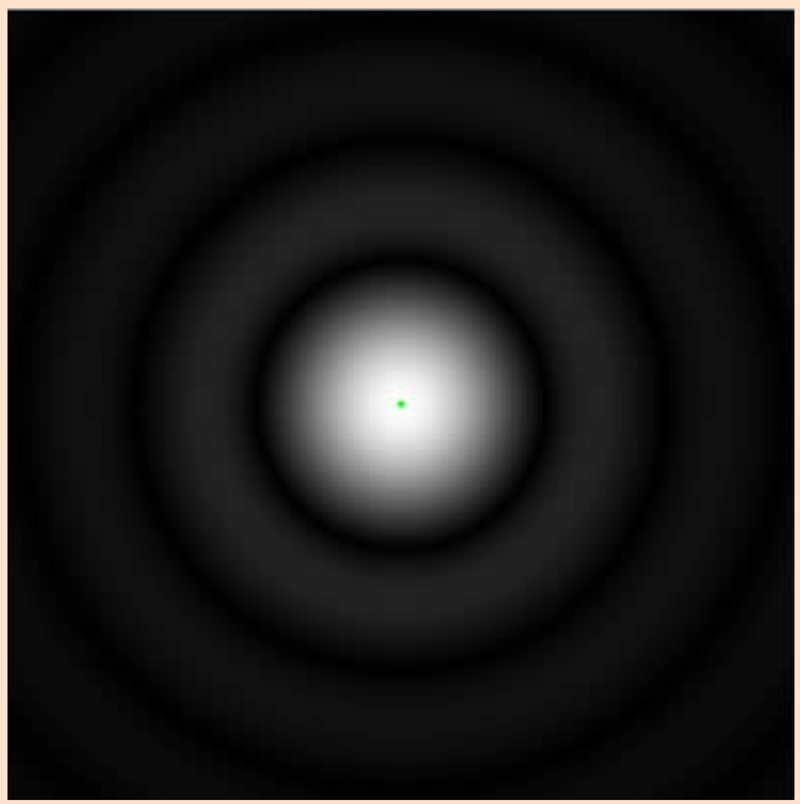
Imaging: Diffraction limit

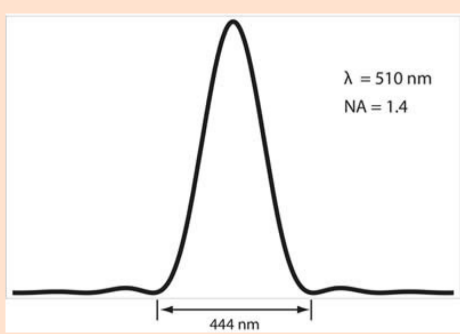
$$r = \frac{0.5\lambda}{NA} = \frac{0.5\lambda}{n\sin\theta}$$



$$r = \frac{0.61\lambda}{NA} = \frac{0.61\lambda}{n\sin\theta}$$





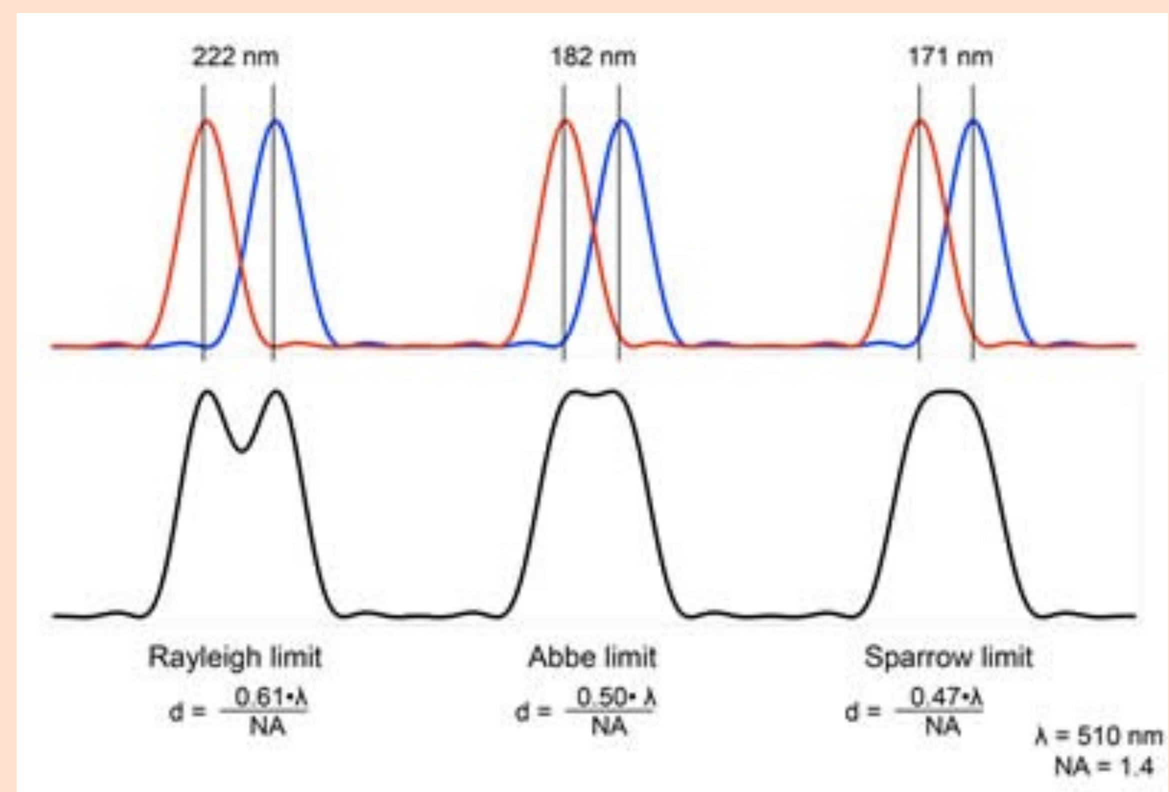


$$diam = 444\ nm$$

GFP: green fluorescent protein (2-4 nm)

$$\lambda = 510nm, \quad NA = 1.4$$

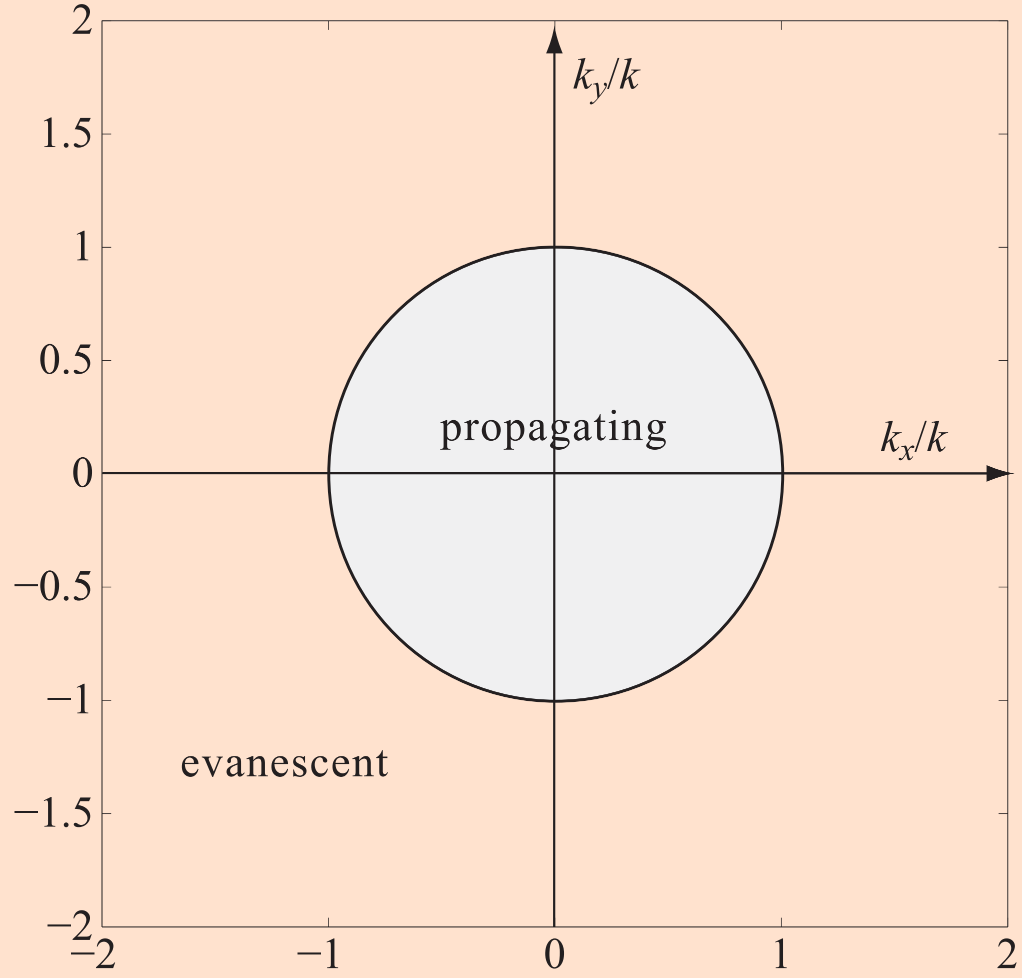
$$r = 222\ nm$$



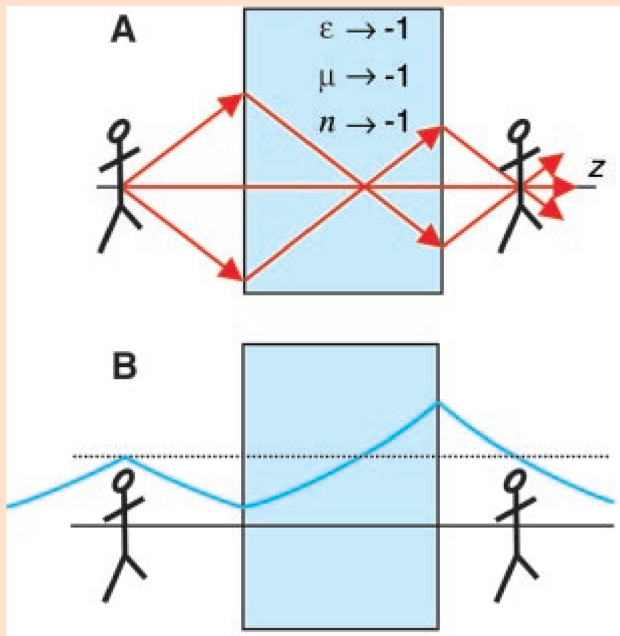
Propagating vs evanescent waves

$$\mathbf{E}(x,y;z) = \int dk_x \int dk_y\, \boldsymbol{\mathcal{E}}(k_x,k_y;0)e^{i(k_x x + k_y y)}e^{\pm i k_z z}$$

$$k_z^2 = k^2 - (k_x^2 + k_y^2)$$



Perfect lensing and imaging
Beyond the diffraction limit

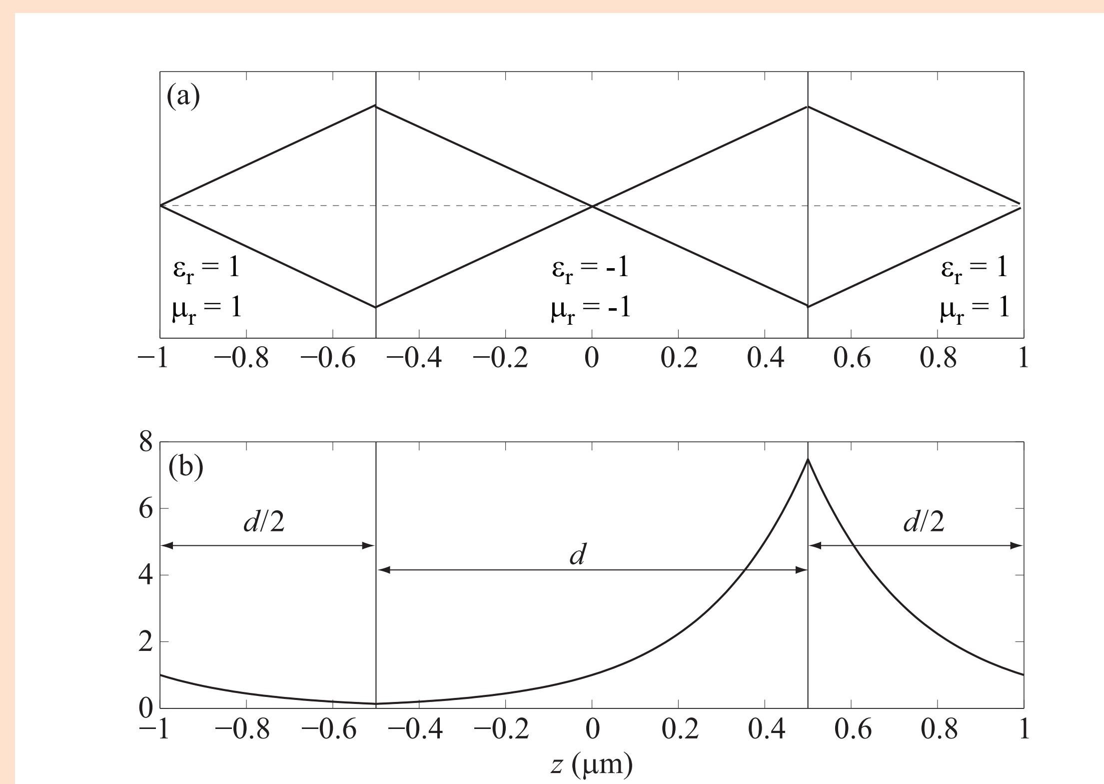


Smith *et al*, *Science* **305**, 788, 2004.

Subwavelength focusing

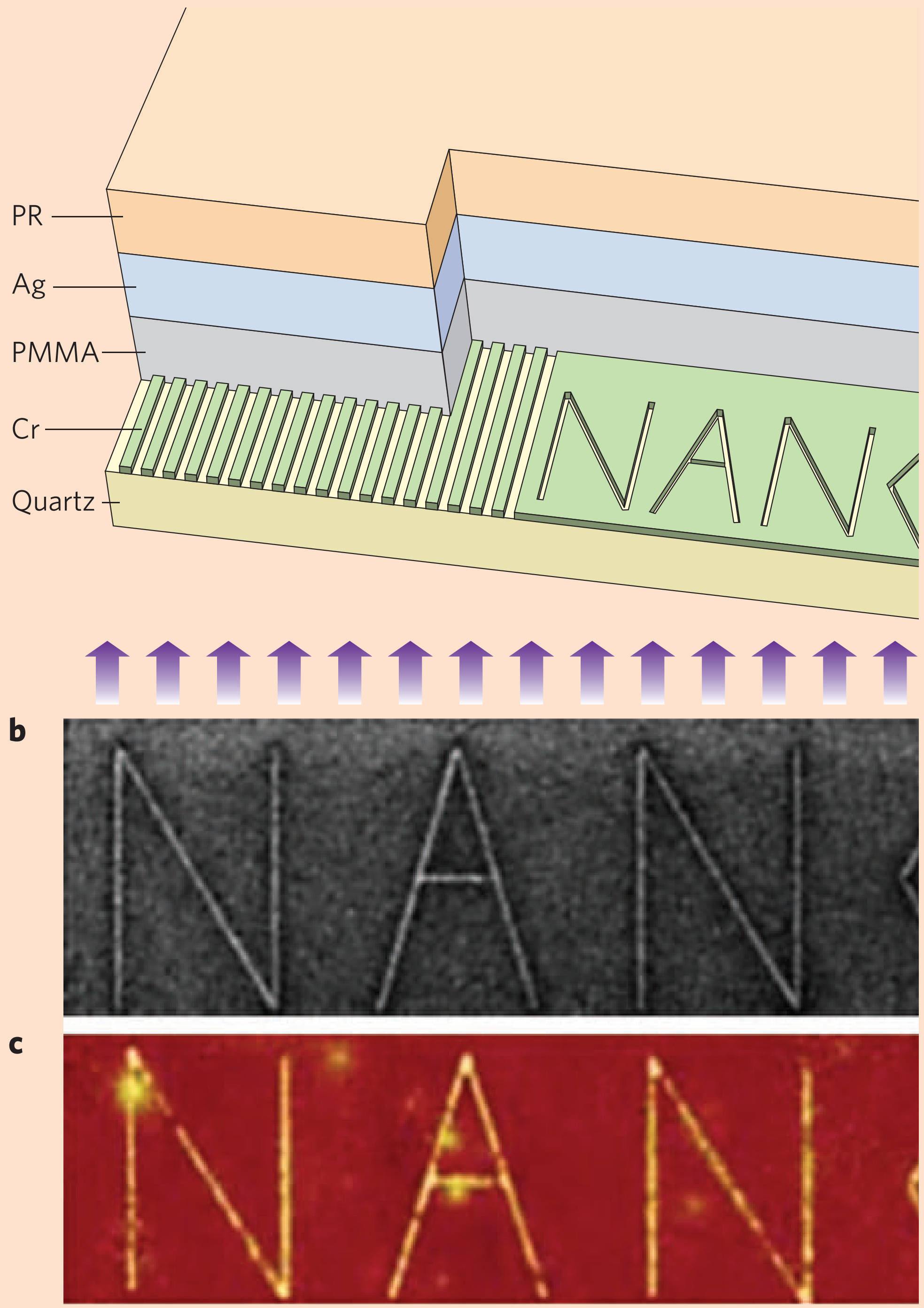
demonstrated in microwave by Grbic *et al*, *Phys. Rev. Lett.*, **92**, 117403, 2004.

Limitation: loss of the system



EIT
Optical nano circuits
Invisibility cloaks

Superlens: proof of principle



Experimental demonstration of subwavelength imaging through a thin silver layer

The sample, inscribed in the form of the word 'NANO' in chromium film, is separated by a thin layer of PMMA from a 35−nm−thick silver film acting as a superlens.

The image is recorded on a photoresist in the form of topographic modulation.

b, A focused ion beam (FIB) image of the inscribed object.

c, AFM image of the topographic modulation corresponding to the near−field image obtained from the superlens.

Science **308,** 534–537 (2005).

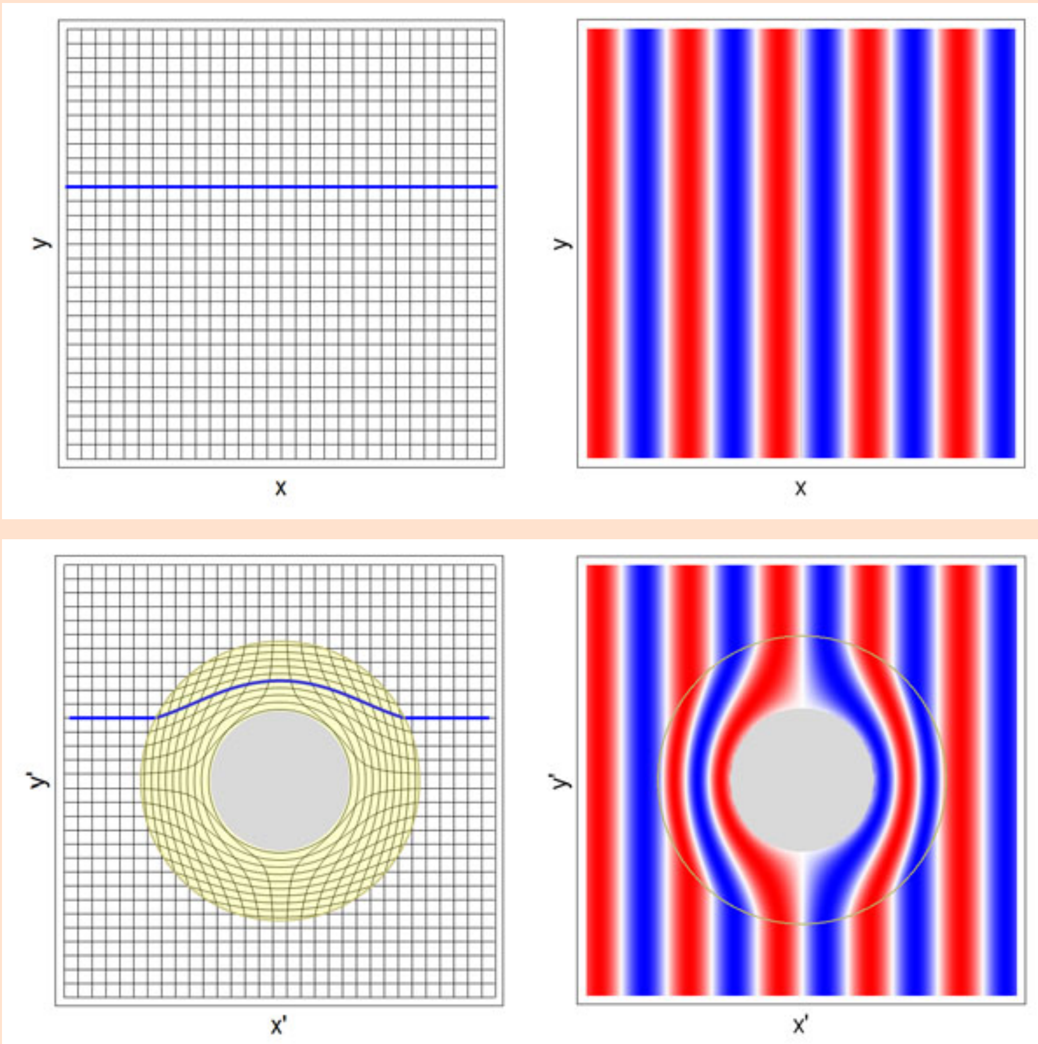
Invisibility cloaks

***H G Wells** The Invisible Man*

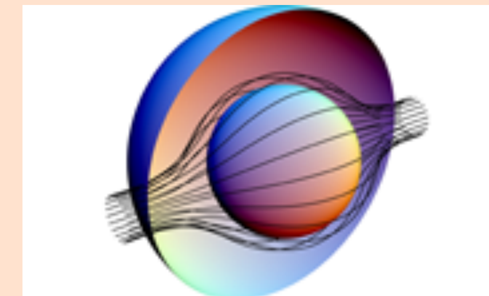
"You make the glass invisible by putting it into a liquid of nearly the same refractive index; a transparent thing becomes invisible if it is put in any medium of almost the same refractive index. And if you will consider only a second, you will see also that the powder of glass might be made to vanish in air, if its refractive index could be made the same as that of air; for then there would be no refraction or reflection as the light passed from glass to air."

"Yes, yes," said Kemp. "But a man's not powdered glass!"

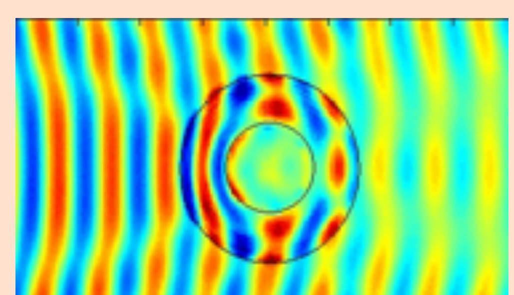
Transformation optics



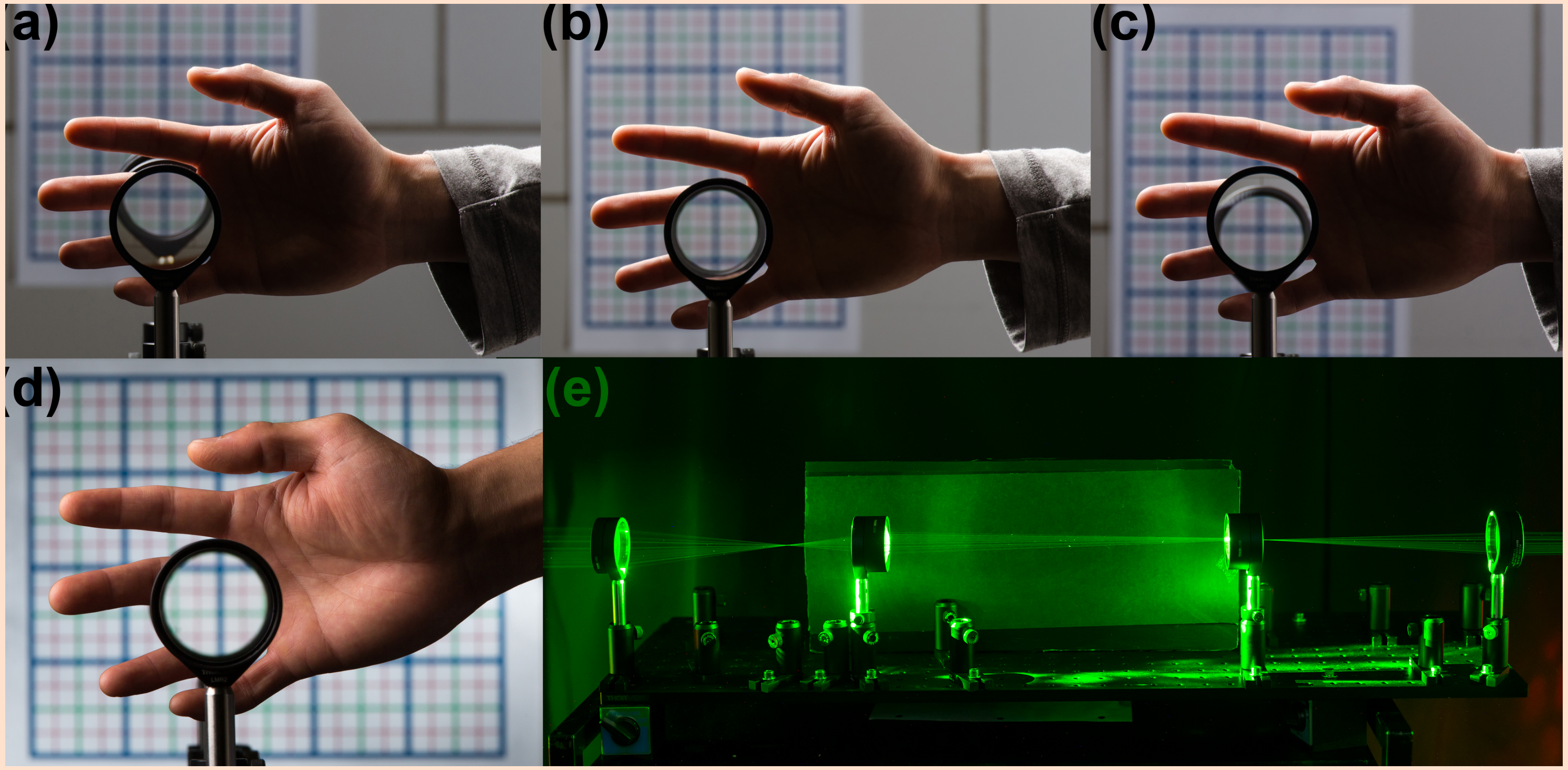
Pendry, Schurig and Smith
Leonhardt
2006

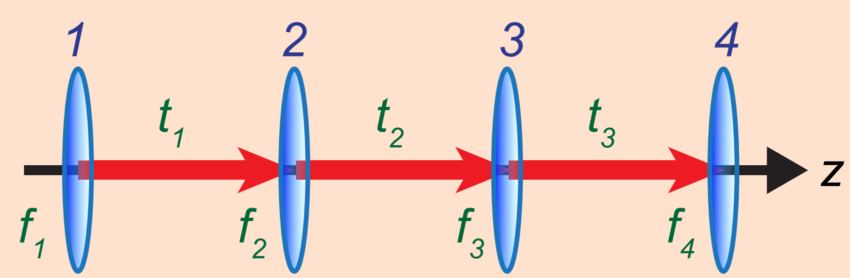


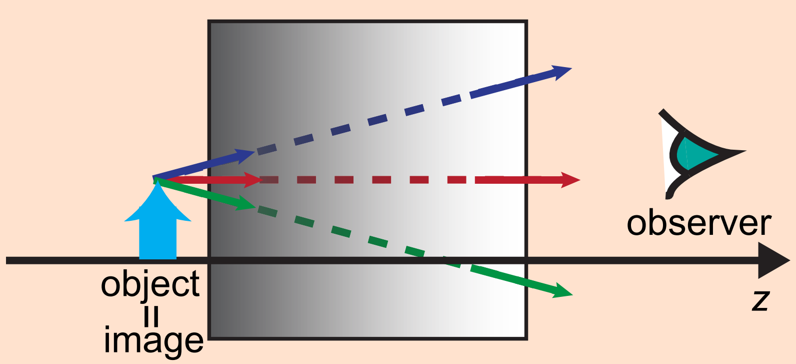
Conformal mapping
curve the optical space



Rochester cloak



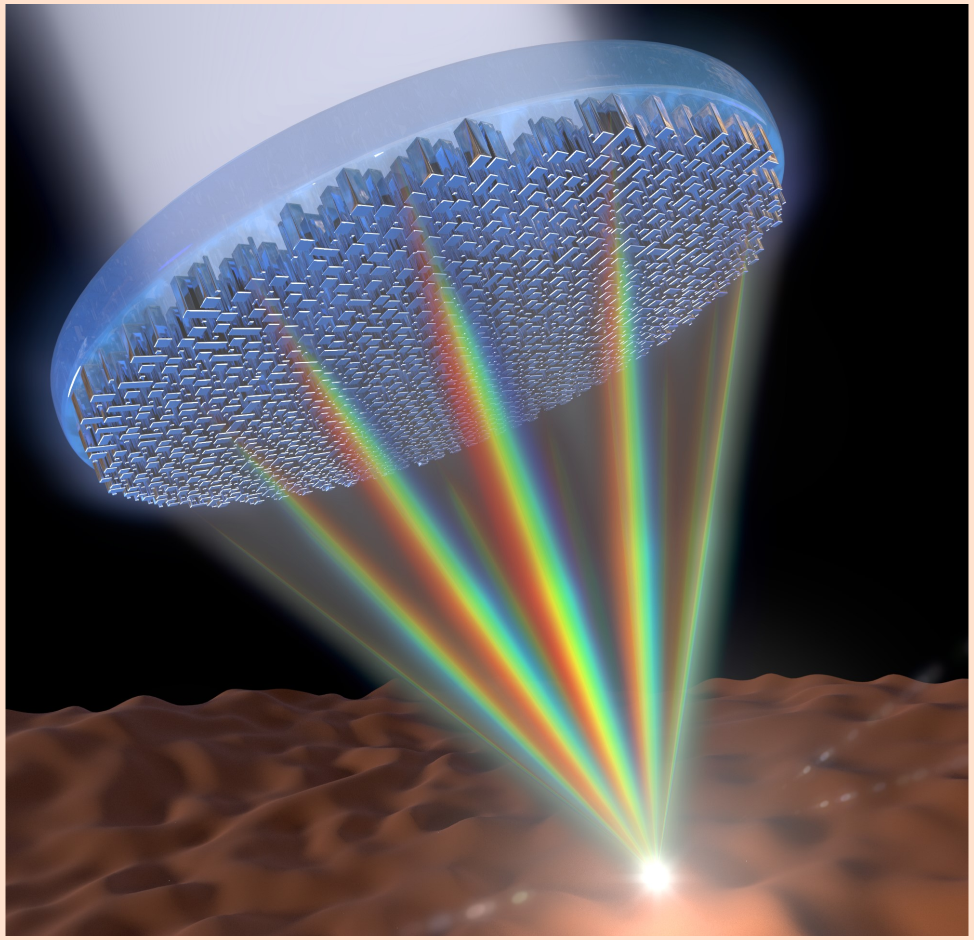




$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{perfect cloak}} = \begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}.$$

J Choi etal, Opt Exp. 2015

Metasurfaces: achromatic lens



Yu, Capasso, Nat. Mat. 2014
Harvard

Conventional optical components

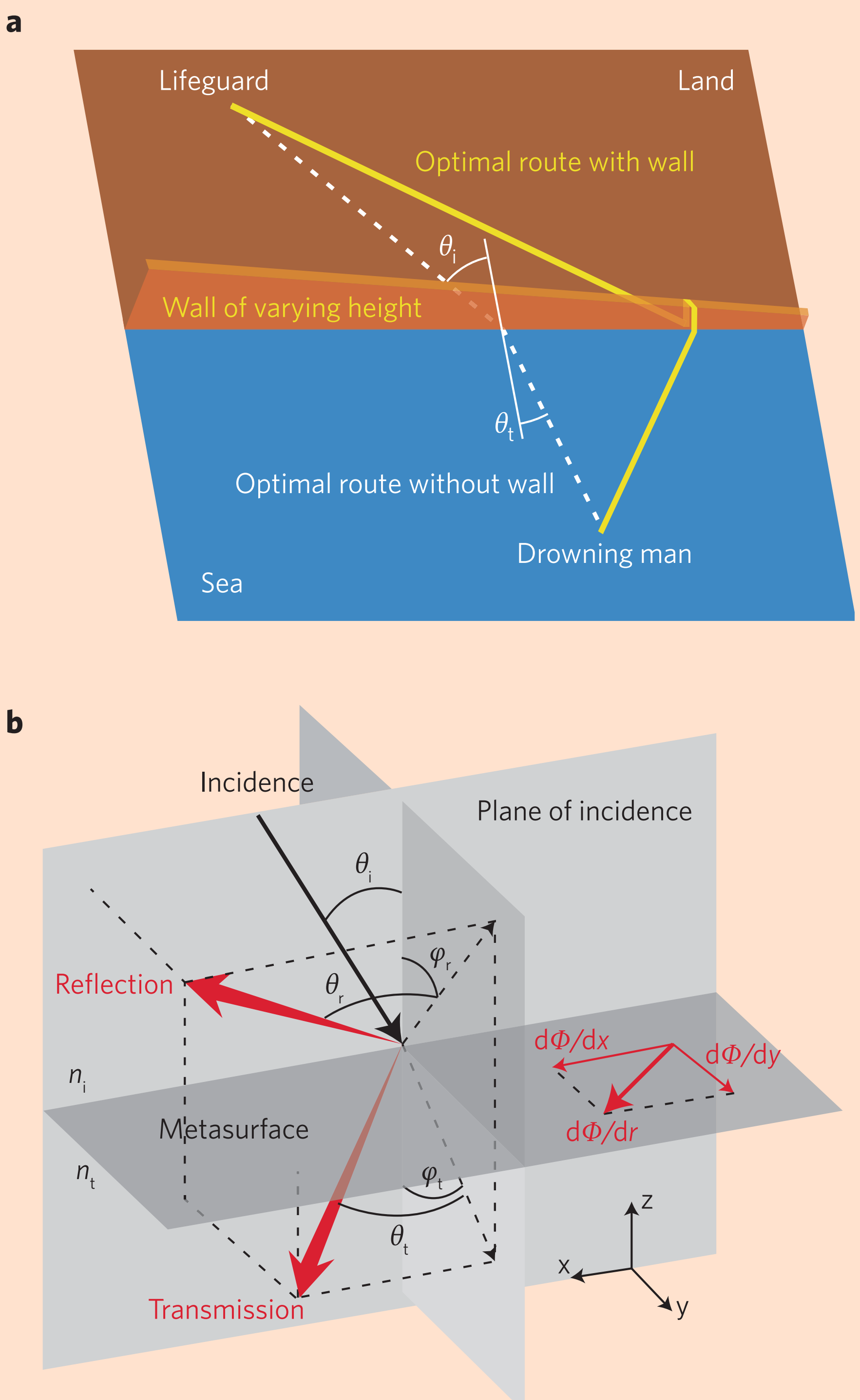
Lenses, waveplates and holograms rely on light propagation over distances much larger than the wavelength to shape wavefronts

Metasurfaces

Ultrathin optical components producing abrupt changes over the scale of wavelength
in the phase, amplitude and/or polarization of a light beam

Arrays of nano antennas with sub-wavelength separation

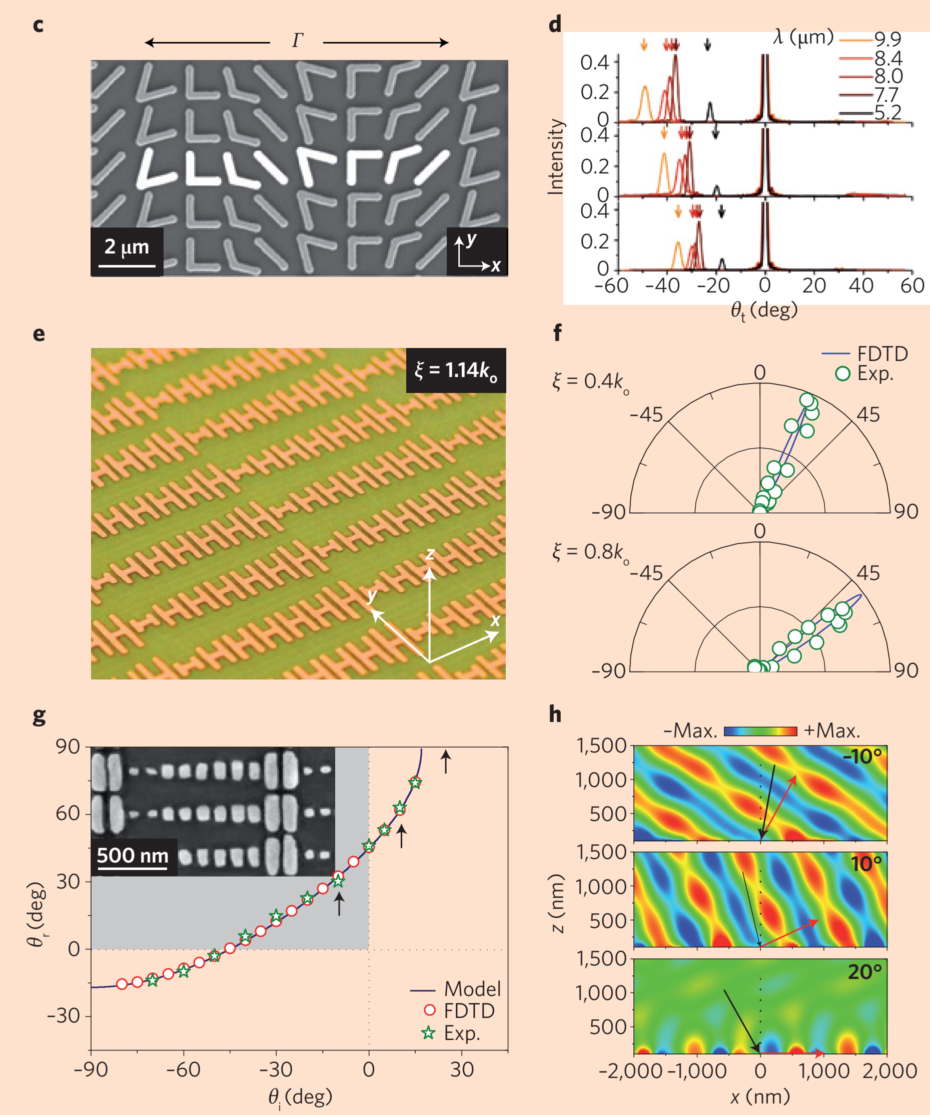
Generalised law of reflection & refraction



$$v_{land} > v_{sea}$$

$$\sin\theta_i/v_{land} - \sin\theta_t/v_{sea} = 0$$

Wall mimics the phase gradient



Capasso

Conclusions

- Theoretical basis of expecting novel phenomena

- Plasmonics: from concepts to experiments

- Applications
  Superlensing and super-resolution
  Electromagnetically induced transparency
  Plasmonic sensing
  Near-field imaging
  Optical invisibility cloaks
  Metasurfaces