Pendry lensing (Chapter 14, Wave Optics)

J. Pendry, 2000, Phys. Rev. Lett. showed that a slab of negative index medium can amplify the evanescent waves.

Thus, earlier

Veselago 1968 : negative slab can focus propagating waves

Pendry 2000 : can focus also evanescent waves.

One thus has the possibility of a perfect lens focusing both propagating and evanescent waves.

Note a standard convex lens focuses only the propagating part.

Weyl representation : em wave (general) has both prop. and evanescent parts

$$\vec{E}(x,y;z) = \int dk_x dk_y \, \underbrace{\vec{\mathcal{E}}(k_x,k_y;z)\, e^{i(k_x x + k_y y)}}_{\text{spatial harmonics}}$$

$$\vec{\mathcal{E}}(k_x,k_y;z) = \frac{1}{4\pi^2}\int dx\, dy\, \vec{E}(x,y;z)\, e^{-i(k_x x + k_y y)}$$

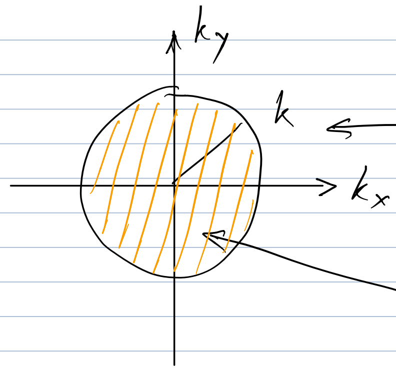
Put in Helmholtz eqn. $\nabla^2 \vec{E} + k^2 \vec{E} = 0$

$$\vec{\mathcal{E}}(k_x,k_y;z) = \vec{\mathcal{E}}(k_x,k_y;0)\, e^{\pm i k_z z} \quad (*)$$

$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$

$$\vec{E}(x,y;z) = \int dk_x\, dk_y\, \mathcal{E}(k_x,k_y;0)\, e^{i(k_x x + k_y y)}\, e^{\pm i k_z z}$$

$\pm$ refer to forward and backward propagating waves.



$k_x^2 + k_y^2 > k^2$

Evanescent (k_z purely imaginary damped solutions)

propagating (k_z real $\rightarrow$ oscillatory solution)

$k_x^2 + k_y^2 \le k^2$

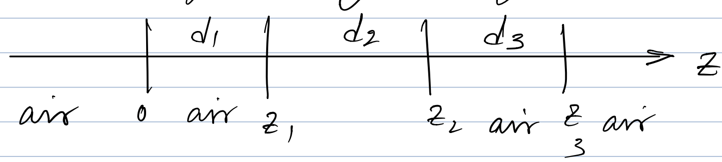
Another important aspect of field propagation ⊛

$$\vec{\mathcal{E}}(k_x, k_y; z) = \mathcal{E}(k_x, k_y; 0)\, e^{\pm i k_z z}$$

$$= \underline{t(k_x, k_y; z)}\, \mathcal{E}(k_x, k_y; 0)$$

<u>Propagator</u> $\longrightarrow$ (transfer function)

Now specialize to 2d (x, z) and a layered medium with central layer being a negative index medium.



$$H_y(x, z = d_1 + d_2 + d_3) = \int dk_x\, t_1(k_x, d_1)\, t_2(k_x, d_2)\, t_3(k_x, d_3)\, H_y(k_x, 0)\, e^{i k_x x},$$

We need to calculate the transfer functions for each layer.

We assumed p-polarized monochrom. wave

$$\vec{k} = (k_x, k_{0z})$$

$$k_x^2 + k_{0z}^2 = \omega^2 \epsilon_0 \mu_0 = k_0^2$$

Propagation from $0 - z_1$ $\qquad t_1 = e^{i k_{0z} d_1}$

$z_2 - z_3$ $\qquad t_3 = e^{i k_{0z} d_3}$

In 2. there are both forward and backward propagating waves.

$$t_2 = \frac{2 p_{0z}}{(m_{11} + m_{12} p_{0z}) p_{0z} + (m_{21} + m_{22} p_{0z})} \qquad p_{0z} = \frac{k_{0z}}{k_0 \epsilon_2}$$

$$= \frac{2 p_{0z}}{\left(\cos k_{2z} d_2 - \frac{i\, p_{0z}}{p_{2z}} \sin k_{2z} d_2\right) p_{0z} + \left(-i p_{2z} \sin k_{2z} d_2 + \cos k_{2z} d_2 \cdot p_{0z}\right)}$$

Let $k_{2z} d_2 = x$.

$$= \frac{2 \times 2}{\left[\left(e^{ix} + e^{-ix}\right) - \frac{\cancel{i}}{\zeta}\left(\frac{e^{ix} - e^{-ix}}{\cancel{i}}\right)\right] + \left[-\cancel{i}\zeta \frac{e^{ix} - e^{-ix}}{\cancel{i}} + \left(e^{ix} + e^{-ix}\right)\right]}$$

$$= \frac{4\zeta}{\left[\zeta\left(e^{ix} + e^{-ix}\right) - \left(e^{ix} - e^{-ix}\right)\right] + \left[\zeta\left(e^{ix} + e^{-ix}\right) - \zeta^2\left(e^{ix} - e^{-ix}\right)\right]}$$

$$\zeta = \frac{p_{2z}}{p_{0z}} = \frac{k_{2z} k_0 \epsilon_0}{k_0 \epsilon_2 k_{0z}} = \frac{\epsilon_0 k_{2z}}{\epsilon_2 k_{0z}}$$

Typo in the book

$$= \frac{4\zeta}{e^{-ix}\left(2\zeta + 1 + \zeta^2\right) + e^{ix}\left(2\zeta - 1 - \zeta^2\right)}$$

$$t_2 = \frac{4\zeta}{(1+\zeta)^2 e^{-i k_{2z} d_2} - (1-\zeta)^2 e^{+i k_{2z} d_2}}$$

Total transfer function

$$t = t_1 t_2 t_3$$

Important : For perfect imaging we need to have a flat transfer function irrespective of the value of the k_x (frequency of the spatial harmonic)
spatial

Let us now specialize to negative material with ideal parameters

$$\epsilon_r = -1, \quad \mu_r = -1 \qquad \epsilon_2 = \epsilon_0 \epsilon_r, \quad \mu_2 = \mu_0 \mu_r$$

Let $d_1 = d_3 = \frac{d}{2}$, $d_2 = d$

then
$$\zeta = \frac{\epsilon_0 k_{2z}}{\epsilon_2 k_{0z}} = \frac{\epsilon_0 \sqrt{k_0^2 \epsilon_2 \mu_2 - k_x^2}}{\epsilon_0 \epsilon_r \sqrt{k_0^2 \epsilon_0 \mu_0 - k_x^2}} = -1$$

$$t_1 = e^{i k_{0z} d/2}, \quad t_3 = e^{i k_{0z} d/2}, \quad t_2 = e^{-i k_{0z} d}$$

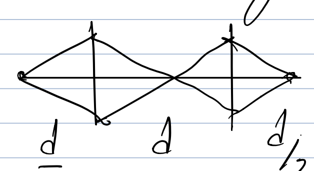
$$1 + \zeta = 0 \qquad 1 - \zeta = 2$$

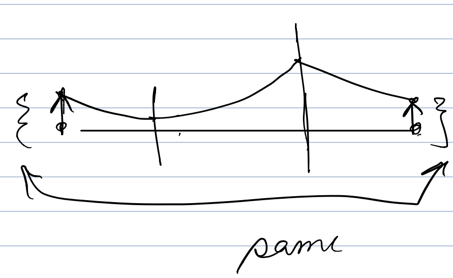
for propagating waves.

$$k_{2z} = \sqrt{k_0^2 \epsilon_2 \mu_2 - k_x^2} = \sqrt{k_0^2 \epsilon_0 \mu_0 - k_x^2} = k_{0z}.$$

$$\Longrightarrow \quad t_1 t_2 t_3 = 1$$

Veselago lensing.





Pendry lensing

$\Downarrow$

$$t_1 = e^{-\varkappa_{0z} d/2} \quad t_2 = e^{+\varkappa_{0z} d} \quad t_3 = e^{-\varkappa_{0z} \frac{d}{2}}$$

For evanescent waves.

$$\varkappa_{0z} = \sqrt{k_x^2 - k_0^2} = -i k_{0z}$$

$$\text{or} \quad k_{0z} = i \varkappa_{0z} \quad \Rightarrow t_1 t_2 t_3 = 1$$

Total transfer function $= 1$ flat for both propagating and evanescent waves

The field emerging at $z = 0$ can be perfectly imaged at $z = z_3 = 2d$. (Fig.)

<u>Question to all.</u> Can there be perfect imaging for a medium with loss?

<u>Assignment</u>

Let me have your thoughts.