Pendry lensing and extraordinary transmission

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Pendry showed that a slab of negative index can amplify the evanescent waves

Earlier, Veselago : negative slab can focus propagating waves

Pendry: can focus also evenscent waves

One thus has the possibility of a perfect lens focusing both propagating and evenscent waves

Note a standard convex lens focuses only the propagating part

Weyl representation: EM wave (general) has both propagating and evanescent part

$$\mathbf{E}(x, y; z) = \int dk_x \int dk_y\, \boldsymbol{\mathcal{E}}(k_x, k_y; z) e^{i(k_x x + k_y y)},$$

→ Spatial harmonic

$$\boldsymbol{\mathcal{E}}(k_x, k_y; z) = \frac{1}{4\pi^2} \int dx \int dy\, \mathbf{E}(x, y; z) e^{-i(k_x x + k_y y)}.$$

Put in the Helmholtz equation, $\nabla^2 \vec{E} + k^2 \vec{E} = 0$

$$\boldsymbol{\mathcal{E}}(k_x, k_y; z) = \boldsymbol{\mathcal{E}}(k_x, k_y; 0) e^{\pm i k_z z}$$

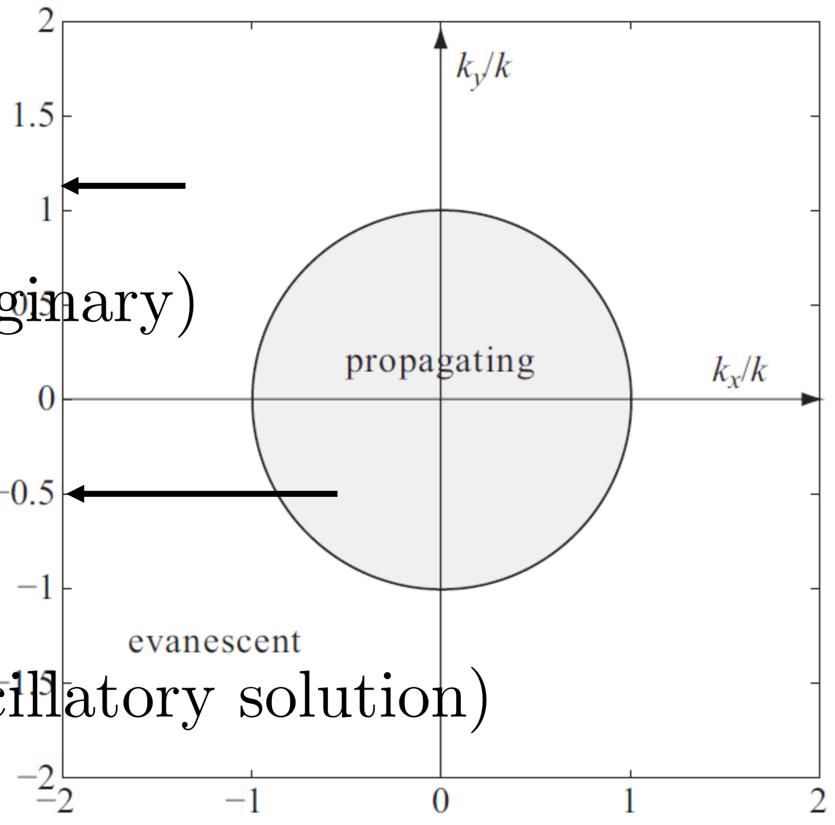
$$k_z = \pm\sqrt{k^2 - (k_x^2 + k_y^2)}.$$

$$\mathbf{E}(x, y; z) = \int dk_x \int dk_y \, \boldsymbol{\mathcal{E}}(k_x, k_y; 0) e^{i(k_x x + k_y y)} e^{\pm i k_z z}.$$

the $\pm$ sign relates to the forward and backward propagating waves.

$k_x^2 + k_y^2 > k^2$

Evanescent (k_z purely imaginary)



$k_x^2 + k_y^2 \leq k^2$

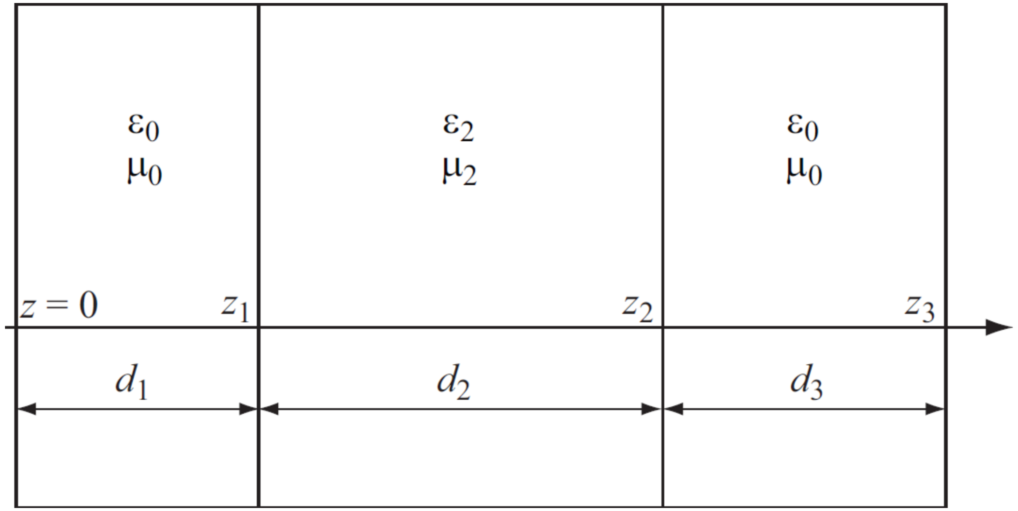
propagating (k_z real $\rightarrow$ oscillatory solution)

Another important aspect of field propagation,

$$\begin{aligned} \boldsymbol{\mathcal{E}}(k_x, k_y; z) &= e^{\pm i k_z z} \boldsymbol{\mathcal{E}}(k_x, k_y; 0) \\ &= t(k_x, k_y, z) \boldsymbol{\mathcal{E}}(k_x, k_y; 0). \end{aligned}$$

↑ Propagator (transfer function)

Now specialize to 2D (x, z) and a layered medium with central layer being a negative index medium.



$$H_y(x, z = d_1 + d_2 + d_3) = \int dk_x t_1(k_x, d_1) t_2(k_x, d_2) t_3(k_x, d_3) \mathcal{H}_y(k_x, 0) e^{ik_x x},$$

We need to calculate the transfer function for each layer

We assume p-polarized monochromatic wave

$$\vec{k} = (k_x, k_{0z}) \qquad k_x^2 + k_{0z}^2 = \omega^2 \epsilon_0 \mu_0 = k_0^2$$

Propagation from $0 - z$ $\qquad t_1 = e^{ik_{0z}d_1}$

$z_2 - z_3$ $\qquad t_3 = e^{ik_{0z}d_3}$

In 2, there are both forward and backward propagating waves

$$t_2 = \frac{2p_{0z}}{(m_{11} + m_{12}p_{0z})p_{0z} + (m_{21} + m_{22}p_{0z})} \qquad p_{0z} = \frac{k_{0z}}{k_0\epsilon_2}$$

$$= \frac{2p_{0z}}{\left(\cos(k_{2z}d_2) - \frac{ip_{0z}}{p_{2z}}\sin(k_{2z}d_2)\right)p_{0z} + (-ip_{2z}\sin(k_{2z}d_2) + \cos(k_{2z}d_2)p_{0z})}$$

$$= \frac{4\zeta}{\left[(e^{ix} + e^{-ix}) - \frac{i}{\zeta}\left(\frac{e^{ix} - e^{-ix}}{i}\right)\right] + \left[-i\zeta\frac{e^{ix} - e^{-ix}}{i} + (e^{ix} + e^{-ix})\right]}$$

Let $k_{2z}d_2 = x$

$$\zeta = \frac{p_{2z}}{p_{0z}} = \frac{k_{2z}k_0\epsilon_0}{k_0\epsilon_2 k_{0z}}$$

$$= \frac{4\zeta}{\left[(e^{ix}+e^{-ix}) - \frac{i}{\zeta}\left(\frac{e^{ix}-e^{-ix}}{i}\right)\right] + \left[-i\zeta\frac{e^{ix}-e^{-ix}}{i} + (e^{ix}+e^{-ix})\right]}$$

$$= \frac{4\zeta}{\left[\zeta(e^{ix}+e^{-ix}) - (e^{ix}-e^{-ix})\right] + \zeta\left[-\zeta(e^{ix}-e^{-ix}) + (e^{ix}+e^{-ix})\right]}$$

$$= \frac{4\zeta}{e^{-ix}(2\zeta+1+\zeta^2) + e^{ix}(2\zeta-1-\zeta^2)}$$

$$= \frac{2\zeta}{e^{-ix}(2\zeta+1+\zeta^2) + e^{ix}(2\zeta-1-\zeta^2)}$$

$$= \frac{4\zeta}{(1+\zeta)^2 e^{-ik_{2z}d_2} - (1-\zeta)^2 e^{ik_{2z}d_2}}$$

Total tranfer function, $t = t_1 t_2 t_3$

Important: For perfect imaging we need to have a flat transfer function irrespective of the value of the k_x (spatial frequency of the spatial harmonic)

Let us now specialize to negative materials with ideal parameters.

$$\epsilon_r = -1 \qquad \epsilon_2 = \epsilon_0 \epsilon_r$$
$$\mu_r = -1 \qquad \mu_2 = \mu_0 \mu_r$$

Let $d_1 = d_3 = \frac{d}{2}$ $\qquad d_2 = d$

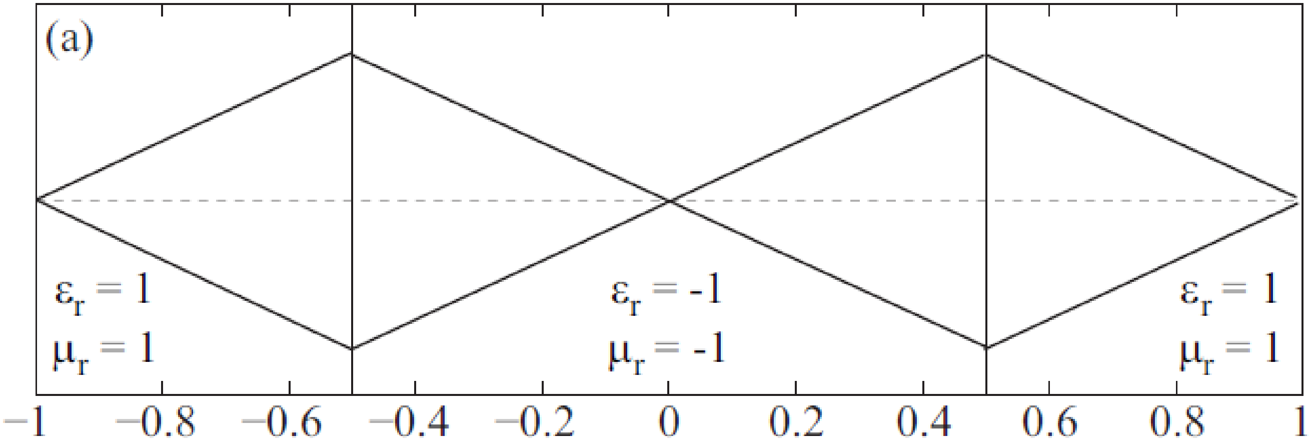
Then, $$\zeta = \frac{\epsilon_0 k_{2z}}{\epsilon_2 k_{0z}} = \frac{\epsilon_0 \sqrt{k_0^2 \epsilon_2 \mu_2 - k_x^2}}{\epsilon_0 \epsilon_r \sqrt{k_0^2 \epsilon_0 \mu_0 - k_x^2}} = -1$$

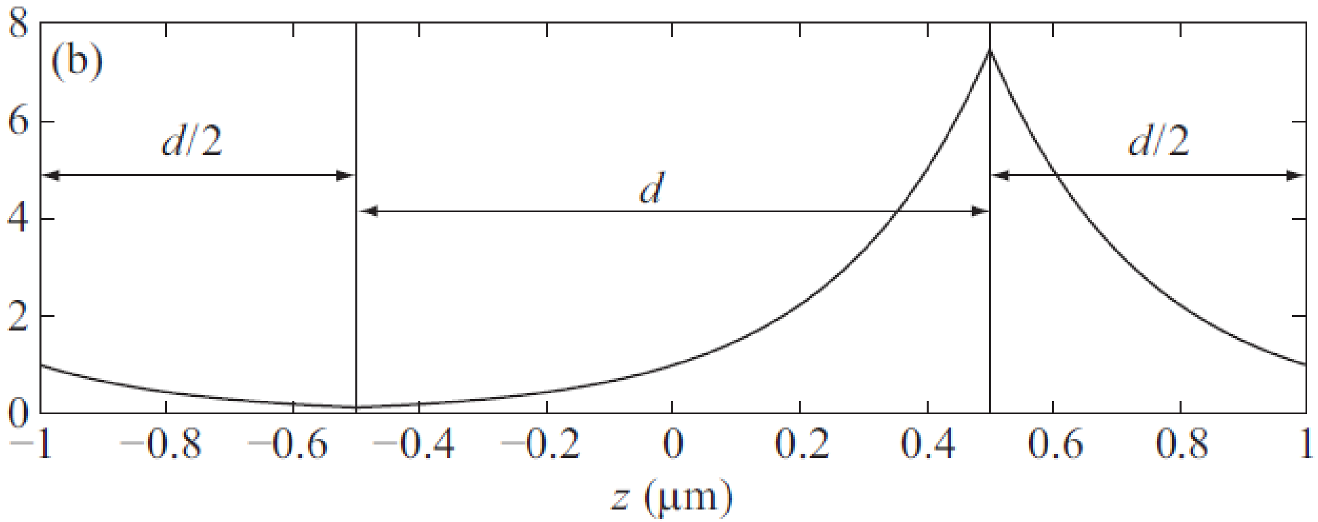
$$t_1 = e^{ik_{0z}d}/2 \qquad t_3 = e^{ik_{0z}d/2} \qquad t_2 = e^{-ik_{0z}d}$$

$$k_{2z} = \sqrt{k_0^2 \epsilon_2 \mu_2 - k_x^2} = \sqrt{k_0^2 \epsilon_0 \mu_0 - k_x^2} = k_{0z}$$

For propagating waves, $t_1 t_2 t_3 = 1$

Veselago lensing





For evanescent waves, $t_1 = e^{-x_{0z} d/2}$ $t_2 = e^{x_{02} d}$ $t_3 = e^{-x_{02} d/2}$

$$x_{02} = \sqrt{k_2^2 - k_0^2} = -ik_{0z} \qquad \text{or } k_{0z} = ix_{0z} \Rightarrow t_1 t_2 t_3 = 1$$

Total transfer function =1 Flat for both propagating and evanescent waves

The field emerging at $z = 0$ can be perfectly imaged at $z = z_3 = 2d$

Extraordinary transmission

Diffraction in the framework of the scalar diffraction theory of Kirchhoff has some severe drawbacks in defining the boundary conditions: It does not give zero tangential component of the electric field on the screen.

For a tiny hole satisfying $a \ll \lambda$ the first approximation result for the normalized-to-area transmission $\mathcal{T}$,

$$\mathcal{T} \approx \frac{64}{27\pi^2}\left(\frac{2\pi a}{\lambda}\right)^4 .$$

The transmission falls off with reducing a. $\Rightarrow$ There will be negligible transmission from subwavelength tiny holes.

When the subwavelength aperture is close to the wavelength: a/λ is not too small. There can be excitation of the localized plasmons and surface plasmons in the metal screen-aperture system.

EOT: The basic structure is a two-dimensional periodic array of sub wavelength holes perforated on a optically thick metal film.

The diffraction of light through such a structure is associated with peaks and dips in the transmission spectra

EOT: Normalized-to-area transmission is more than unity. EOT phenomena is linked to the excitation of the surface plasmons and localised plasmons.

The periodicity in the plane of the metal screen with the holes provide for the missing momentum mismatch

$\Rightarrow$ SP's can be excited on both the faces of the metal screen. The spectral features arising in the transmission spectra are due to these excitations.

Two classes for EOT:

(i) Bull's eye: A single hole surrounded by periodic grooves on both sides of the film. The momentum mismatch is compensated by the periodicity of the grooves. EOT is mediated when light at the exit face can couple to the modes of the periodic grooves. The modes can then couple to the free space light which interferes with the light traveling directly through the hole.

$\Rightarrow$ For an incident collimated beam, the output light can be highly collimated, with counterintuitive focusing.

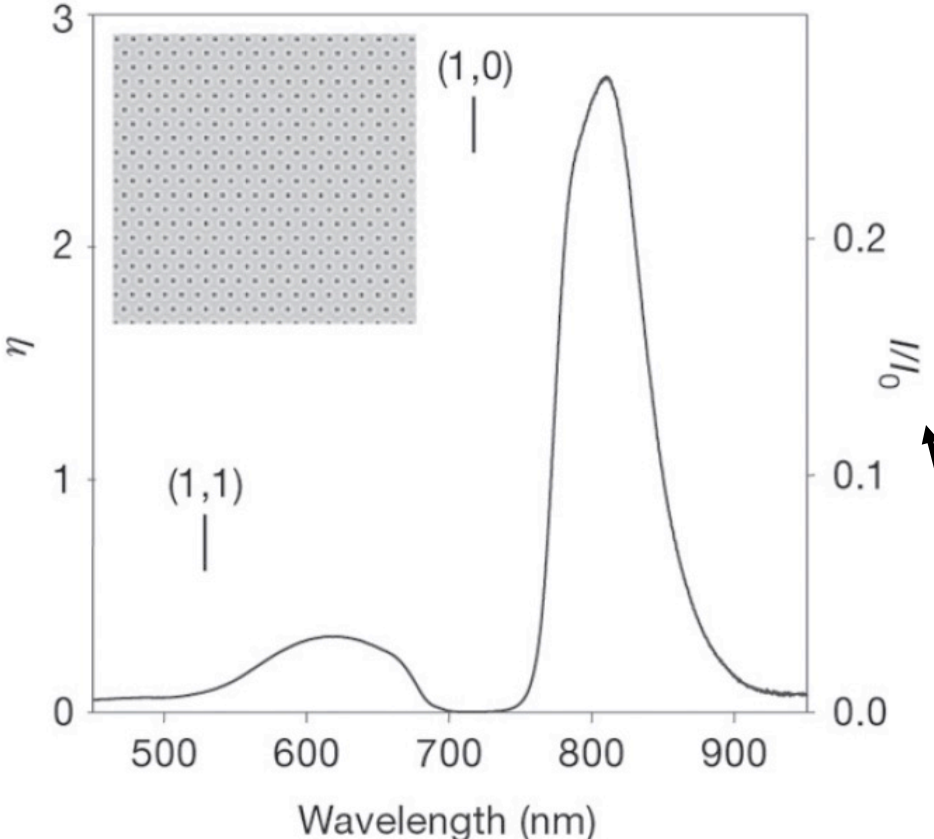
Standard diffraction theory: The narrower the hole, the larger is the beam angular divergence.

The peak occurs when surface supports the standing surface plasmon waves.

(a) The incident light excites the SP's on the front surface.

(b) The transmission through the holes to the second surface

(c) Reemission from the exit surface.



Transmission spectrum of hole arrays for the triangular hole array

hole diameter 170 nm, period 520 nm

normalized to the area (occupied by the holes) transmission.

The peak occurs when surface supports the standing surface plasmon waves.

An estimate of the wavelength λ_m where the peak occurs can be made by looking at the momentum matching condition for a two-dimensional grating. For normal incidence the (i, j)-th resonance peak location is given by

$$\lambda_m = \frac{\Lambda}{\sqrt{\frac{4}{3}(i^2 + ij + j^2)}} \sqrt{\frac{\varepsilon_m \varepsilon}{\varepsilon_m + \varepsilon}},$$

where Λ is the grating period and ε_m (ε) is the dielectric function of the metal (dielectric). This does not incorporate the effect of holes and the associated losses and hence the results predicted are blue-shifted from the observed data.