Nano Optics and Plasmonics: Theory
Lecture 7

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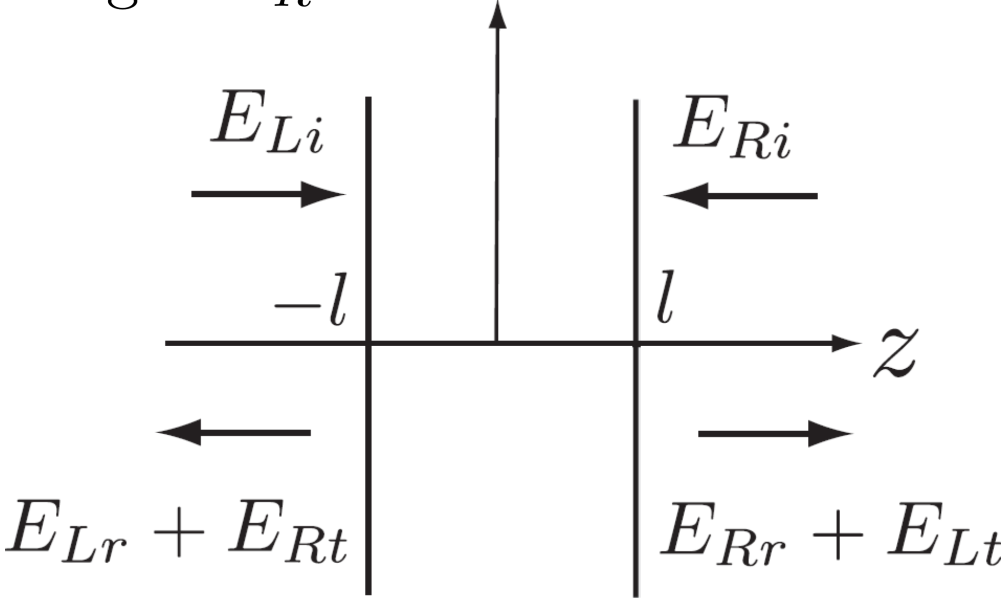
Nonreciprocity in reflection from stratified media

A given one-dimensional system is said to exhibit reciprocity if the scattering is insensitive to whether it is excited from left or from right.

<u>General reciprocity relations for an arbitrary linear stratified medium</u>

Consider a stratified medium embedded in vacuum occupying $-l \leq z \leq l$ having a scalar complex dielectric function $\varepsilon(z)$.

The solution for fields incidence from left E_L and right E_R can be written as



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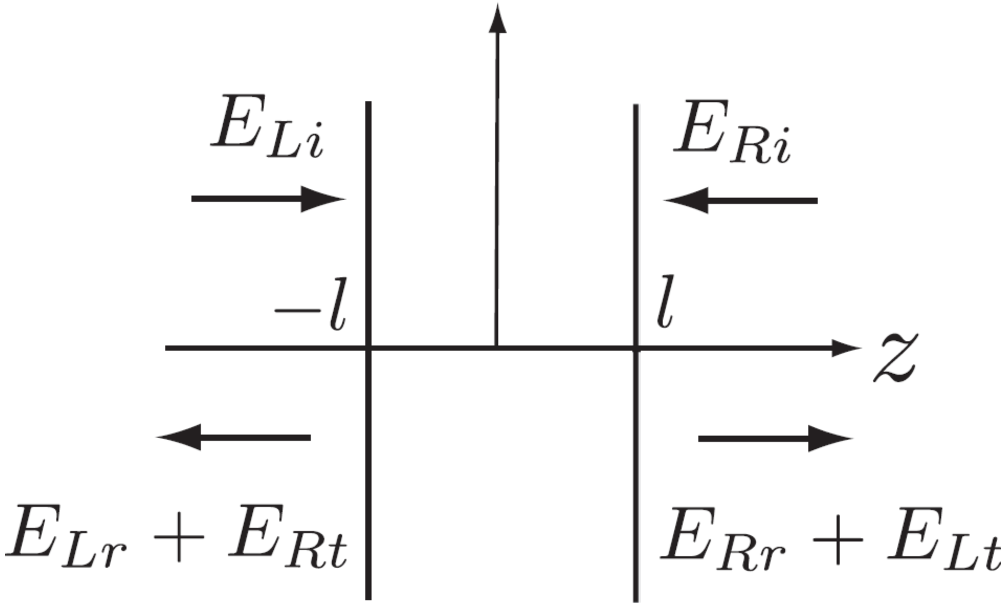
<u>General reciprocity relations for an arbitrary linear stratified medium</u>

Consider a stratified medium embedded in vacuum occupying $-l \leq z \leq l$ having a scalar complex dielectric function $\varepsilon(z)$.

The solution for fields incidence from left E_L and right E_R can be written as

$$\begin{aligned} E_L &= E_{Li}e^{ik_0 z} + E_{Lr}e^{-ik_0 z}, \quad z \leq -l, \\ &= E_{Lt}e^{ik_0 z}, \quad z \geq l, \\ E_R &= E_{Ri}e^{-ik_0 z} + E_{Rr}e^{ik_0 z}, \quad z \geq l, \\ &= E_{Rt}e^{-ik_0 z}, \quad z \leq -l, \end{aligned}$$

i, r, t- incident, reflected and the transmitted waves



From Helmholtz equations for E_L and E_R^*,

$$\frac{d^2 E_L}{dz^2} + \epsilon(z) k_0^2 E_L = 0 \Rightarrow E_R^* \frac{d^2 E_L}{dz^2} + \epsilon(z) k_0^2 E_R^* E_L = 0$$

$$\frac{d^2 E_R^*}{dz^2} + \epsilon^*(z) k_0^2 E_R^* = 0 \Rightarrow E_L \frac{d^2 E_R^*}{dz^2} + \epsilon^*(z) k_0^2 E_L E_R^* = 0$$

$$\Rightarrow \int_{-l}^{l} \left(E_R^* \frac{d^2 E_L}{dz^2} - E_L \frac{d^2 E_R^*}{dz^2} \right) dz + k_0^2 \int_{-l}^{l} E_L E_R^* (\varepsilon - \varepsilon^*) dz = 0,$$

Using integration by parts,

$$\left[E_R^*\frac{dE_L}{dz}-E_L\frac{dE_R^*}{dz}\right]_{-l}^{l}+k_0^2\int_{-l}^{l}E_LE_R^*(\varepsilon-\varepsilon^*)dz=0,$$

$$\begin{aligned} E_L &= E_{Li}e^{ik_0z}+E_{Lr}e^{-ik_0z}, \quad z\leq -l, \\ &= E_{Lt}e^{ik_0z}, \quad z\geq l, \\ E_R &= E_{Ri}e^{-ik_0z}+E_{Rr}e^{ik_0z}, \quad z\geq l, \\ &= E_{Rt}e^{-ik_0z}, \quad z\leq -l, \end{aligned}$$

Substituting from the solutions to the fields at $z=\pm l$,

$$2ik_0(E_{Lt}E_{Rr}^*+E_{Rt}^*E_{Lr})+2ik_0^2\int E_L(z)E_R^*(z)\,[\mathrm{Im}\ \varepsilon(z)]\,dz=0$$

$$\Rightarrow \qquad (E_{Lt}E_{Rr}^*+E_{Rt}^*E_{Lr})+k_0\int E_L(z)E_R^*(z)\,[\mathrm{Im}\ \varepsilon(z)]\,dz=0$$

For identical fields $E_L = E_R = E$,

$$\left[E^*\frac{dE}{dz} - E\frac{dE^*}{dz}\right]_{-l}^{l} + 2ik_0^2\int_{-l}^{l}|E|^2\text{Im }\varepsilon(z)\,dz = 0.$$

Writing E as,

$$\begin{aligned} E &= E_i e^{ik_0 z} + E_r e^{-ik_0 z}, \quad z \le -l, \\ &= E_t e^{ik_0 z}, \quad z \ge l \end{aligned}$$

Proceeding as before,

Standard optical theorem:

$$|E_t|^2 + |E_r|^2 + k_0\int |E(z)|^2\ \text{Im }\varepsilon(z)dz = |E_i|^2.$$

For reciprocity relations for the transmitted amplitudes, we combine the Helmholtz equations for E_L and E_R in the form

$$\int_{-l}^{l} \left(E_R \frac{d^2 E_L}{dz^2} - E_L \frac{d^2 E_R}{dz^2} \right) dz = 0,$$

which after integration by parts can be simplified to

$$\left(E_R \frac{dE_L}{dz} - E_L \frac{dE_R}{dz} \right)\Bigg|_{-l}^{l} = 0.$$

Making use of the fields E_r and E_L at $\pm l$,

$$E_{Ri} E_{Lt} = E_{Li} E_{Rt},$$

Assume the incident fields from left and right to be the same having unity amplitudes, i.e., $E_{Li} = E_{Ri} = 1$,

$$\Rightarrow E_{Lt} = E_{Rt}$$

This derivation had no reference to form of the absorption present in the medium

$\Rightarrow$ Valid for arbitrary lossy stratified medium.

Thus, transmission is always reciprocal

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$$E_{Lt}E^*_{Rr} + E^*_{Rt}E_{Lr} + k_0 \int E_L(z)E^*_R(z)\ [\mathrm{Im}\ \varepsilon(z)]dz = 0$$

$$\Rightarrow \quad \frac{E^*_{Rr}}{E^*_{Rt}} + \frac{E_{Lr}}{E_{Lt}} + \frac{k_0}{E_{Lt}E^*_{Rt}} \int E_L(z)E^*_R(z)\ \mathrm{Im}\ \varepsilon(z)dz = 0.$$

In absence of absorption ($\mathrm{Im}(\varepsilon) = 0$),

$$E_{Lt} = E_{Rt}, \quad \frac{E^*_{Rr}}{E^*_{Rt}} + \frac{E_{Lr}}{E_{Lt}} = 0, \quad |E_{Lt}|^2 + |E_{Lr}|^2 = 1$$

$\Rightarrow$ leads to the important result

$$|E_{Rr}|^2 = |E_{Lr}|^2, \quad \text{though} \quad E_{Rr} \neq E_{Lr}.$$

$\Rightarrow$ There may not be reciprocity in reflected amplitudes, since they can differ in phase.

Spatial symmetry plays an important role here.

Consider a medium with $\varepsilon(z) = \varepsilon(-z)$.

$\Rightarrow$ If $E(z)$ is a solution of the Helmholtz equation, so is $E(-z)$. Let the incident fields also be symmetric, i.e., $E_i(z) = E_i(-z)$.

The total field $E(z)$ would then satisfy $E(z) = E(-z)$ everywhere.

$$
\begin{aligned}
E(z) &= e^{ik_0 z} + E_{Lr} e^{-ik_0 z} + E_{Rt} e^{-ik_0 z}, \quad z \leq -l, \\
&= e^{-ik_0 z} + E_{Rr} e^{ik_0 z} + E_{Lt} e^{ik_0 z}, \quad z \geq l.
\end{aligned}
$$

$$E(z) = E(-z) \Rightarrow \quad E_{Lr} + E_{Rt} = E_{Rr} + E_{Lt}.$$

Reciprocity of the reflected amplitudes

$$E_{Lt} = E_{Rt} \Rightarrow \qquad E_{Lr} = E_{Rr}.$$

$\Rightarrow$ Spatial symmetry ensures the reciprocity of the reflected amplitudes irrespective of the absorption in the system.

In lossless structures, intensity reflection is reciprocal.

Broken spatial symmetry leads to non reciprocity of phases in the reflected waves.

Nonreciprocity in phases in reflected light

In absence of absorption ($\mathrm{Im}(\varepsilon(z)) = 0$), assuming $E_{r,t} \sim |E_{r,t}| e^{i\phi_{r,t}}$,

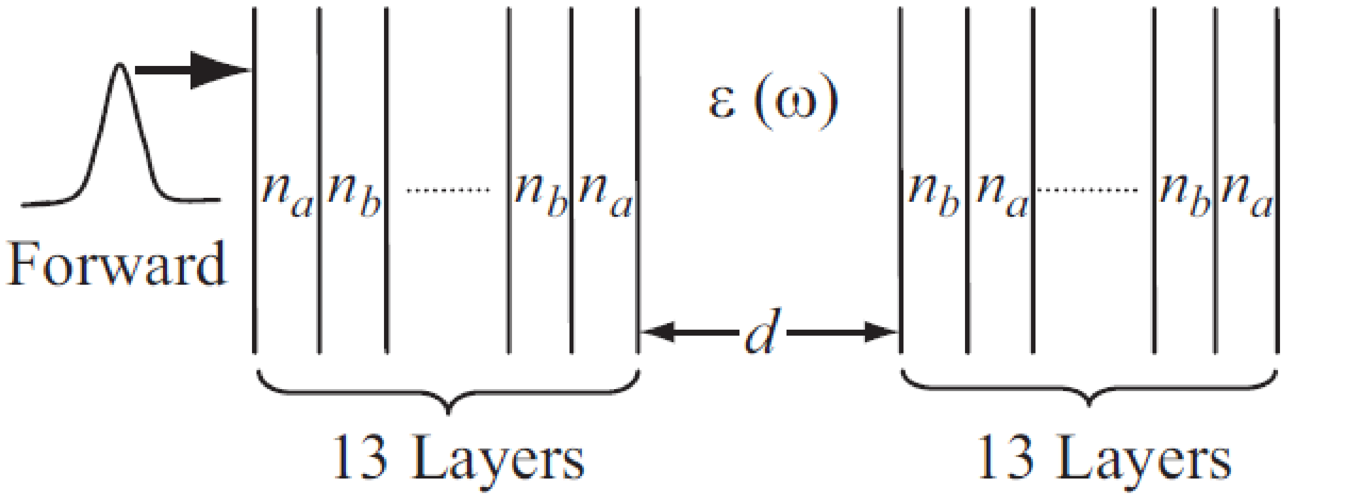
$$\begin{aligned} E_{Lt} &= E_{Rt} \\ \frac{E^*_{Rr}}{E^*_{Rt}} + \frac{E_{Lr}}{E_{Lt}} &= 0, \end{aligned} \quad \Rightarrow \quad \begin{aligned} |E_{Lr}| &= |E_{Rr}|, \\ 2\phi_t &= \phi_{Lr} + \phi_{Rr} + \pi \end{aligned}$$

$$\Rightarrow \tau_t = \frac{(\tau_{Lr} + \tau_{Rr})}{2}.$$

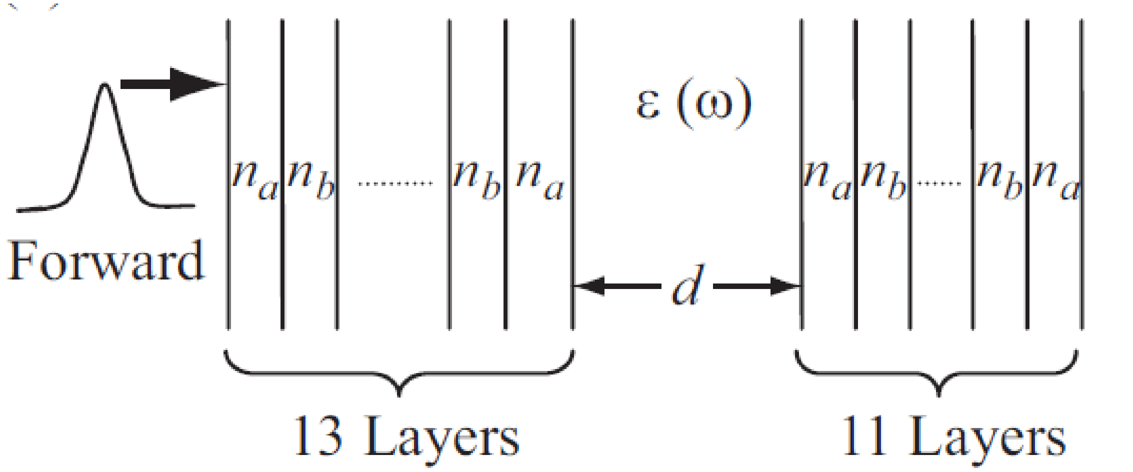
Thus, for a lossless structure lacking inversion symmetry there is no nonreciprocal effect in the intensity reflection coefficient, pulse delays can reveal the asymmetry of the structure.

For example, (for vanishingly small transmission delay τ_t) if the reflected pulse acquires a delay (meaning subluminal reflection) for incidence from left, the same will be superluminal for incidence from the other side.

Pulse transmission and reflection from a symmetric and asymmetric Fabry-Perot cavities



Symmetric FP cavity. The DFB mirrors have 13 alternating high and low index (n_a=2.4, n_b=1.3, respectively) $\lambda/4$ layers.



Asymmetric FP cavity. The left (right) mirror has 13 (11) layers.

The length of each cavity is $d = 6.35\,\mu$m. 'Forward' and 'backward' directions imply incidence of the pulse from left or right.

The DFB mirrors are used so as to lead to high mirror reflections.

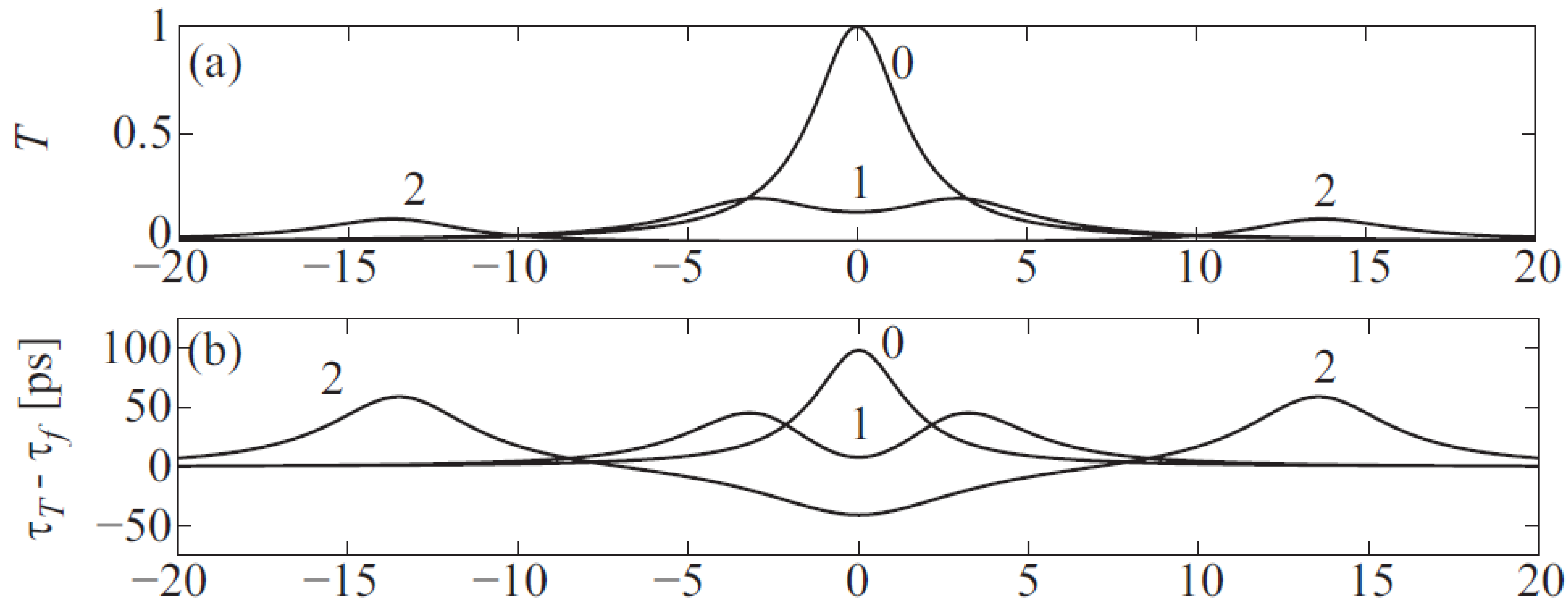
System: Symmetric and asymmetric Fabry-Perot (FP) cavities with or without resonant absorbers with distributed feedback (DFB) mirrors.

The resonant absorbers are modeled by a dielectric function $\varepsilon(\omega)$ given by,

$$\varepsilon(\omega) = \varepsilon_0 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - 2i\gamma\omega}.$$

The plasma frequency ω_p is related to the number density and the dipole matrix elements of the atoms.

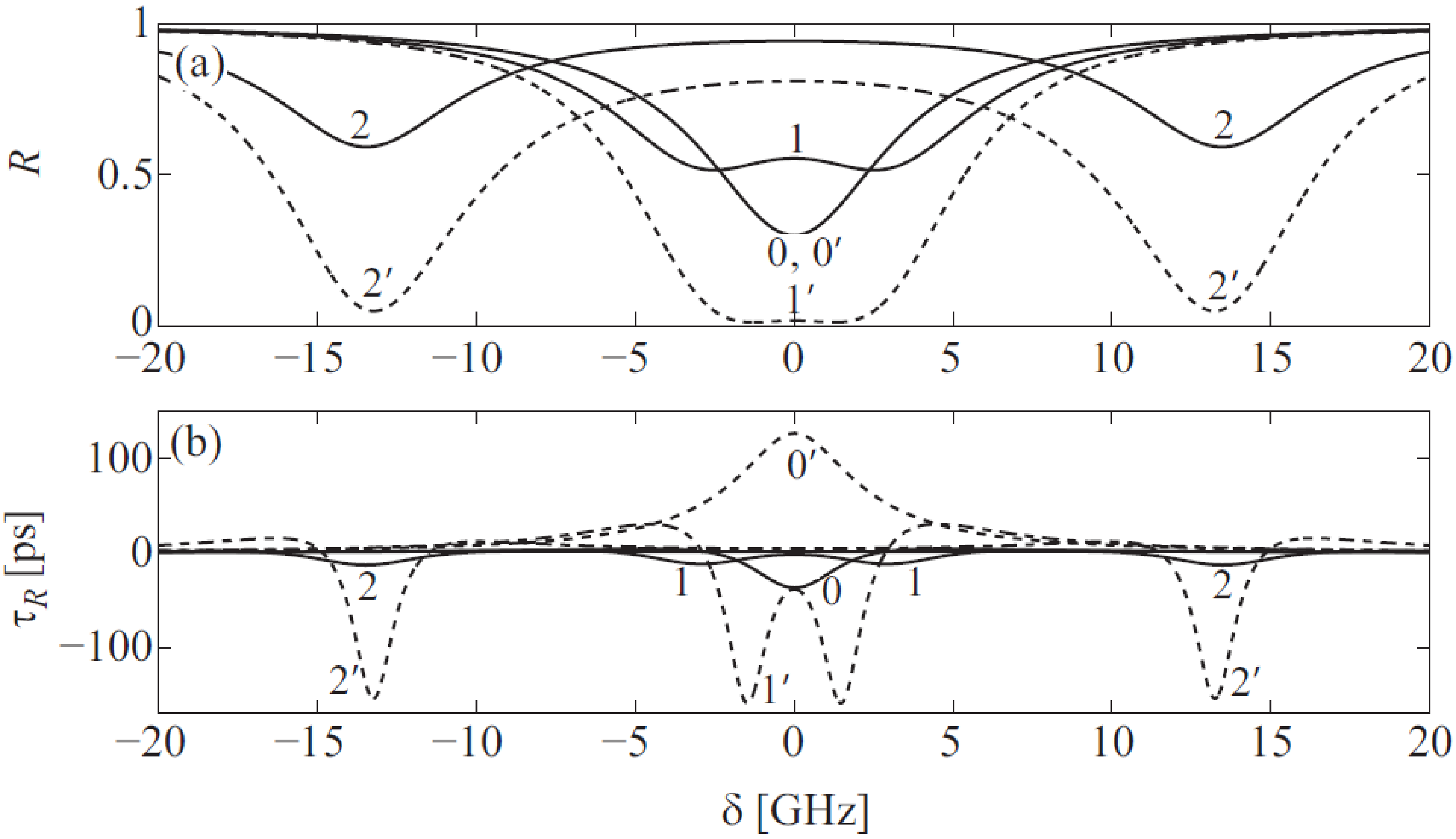
Symmetric FP cavity



The curves with label 0,1 and 2 are for $\omega_p^2/\omega_0^2 = 0.0, 0.08 \times 10^{-9}$ and 1.5×10^{-9}, respectively with $2\gamma/\omega_0 = 1.0 \times 10^{-5}$.

Transmission is reciprocal irrespective of whether the structure under consideration lacks any spatial symmetry. The reciprocity of transmission is insensitive to presence or absence of absorption as well.

Asymmetric FP cavity



Forward (solid) and backward (dashed) intensity reflection coefficient R and corresponding phase time τ_R as functions of detuning δ.

The curves with label $(0, 0')$, $(1, 1')$ and $(2, 2')$ are for $\omega_p^2/\omega_0^2 = 0.0, 0.08 \times 10^{-9}$ and 1.5×10^{-9}, respectively.

The reflection coefficient of an empty asymmetric cavity is the same for both forward and backward directions.

Phase times are not identical for forward (curve marked by 0) and backward (marked by $0'$) directions.

When atoms are introduced in the cavity the degeneracy in the reflection coefficient for forward and backward directions is lifted leading to more pronounced dip in the backward reflection coefficients for larger densities.

$\Rightarrow$ Broken inversion symmetry alone is not adequate to lead to nonreciprocity in reflected amplitudes, while it is sufficient to lead to different phases for incidence from opposite directions.

It is essential for the system to be lossy in order to lead to nonreciprocity in reflected amplitudes.