Exploring the route from leaky Berreman modes to bound states in continuum

Subhasish Dutta Gupta

University of Hyderabad, TIFR Hyderabad, IISER, Kolkata

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With Ghanasyam Remesh, Pravin Vaity and V. G. Achanta

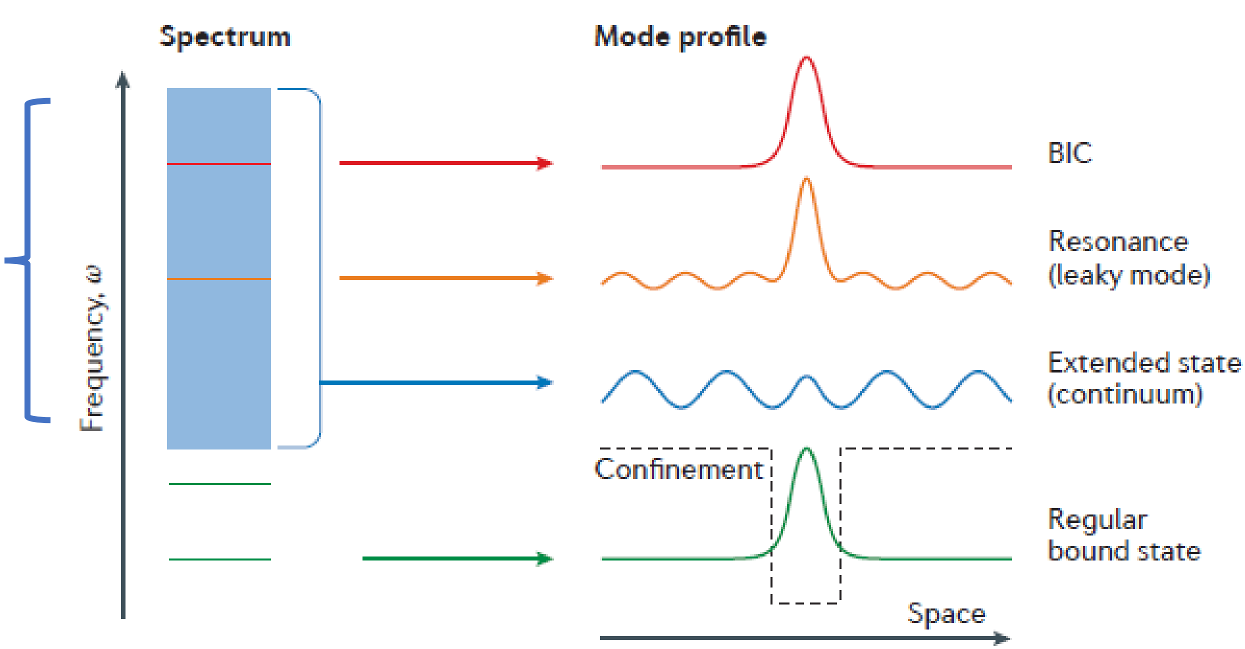
Bound states to BIC

Bound States: Usually frequency outside the continuum
$\Rightarrow$ no path to radiate away

Ex: Bound states of electron : -ve energy.
Continuum : +ve energy.

BICs are exceptions to this! Bound even when energy inside the continuum .

Continuum of energy



$\omega = \omega_0 + i\gamma$

$\omega = \omega_0$

(confined)

Hsu *et. al.,* Nature Rev. Mater 1 (2016) 16048

The first BIC proposed: via potential engineering

Choose ψ and E, find V

$$-\frac{1}{2}\nabla^2\psi + V\psi = E\psi \to V = E + \frac{\nabla^2\psi}{2\psi} \qquad E > 0,\ V \to 0 \text{ at } r \to \infty$$

$$V = ?$$

Example: $\quad$ Let $\psi(\boldsymbol{r}) = f(r)\dfrac{\sin(kr)}{kr}$ $\quad \Big(\psi(\boldsymbol{r}) \to 0$ ar $r \to \infty$, hence bound$\Big)$

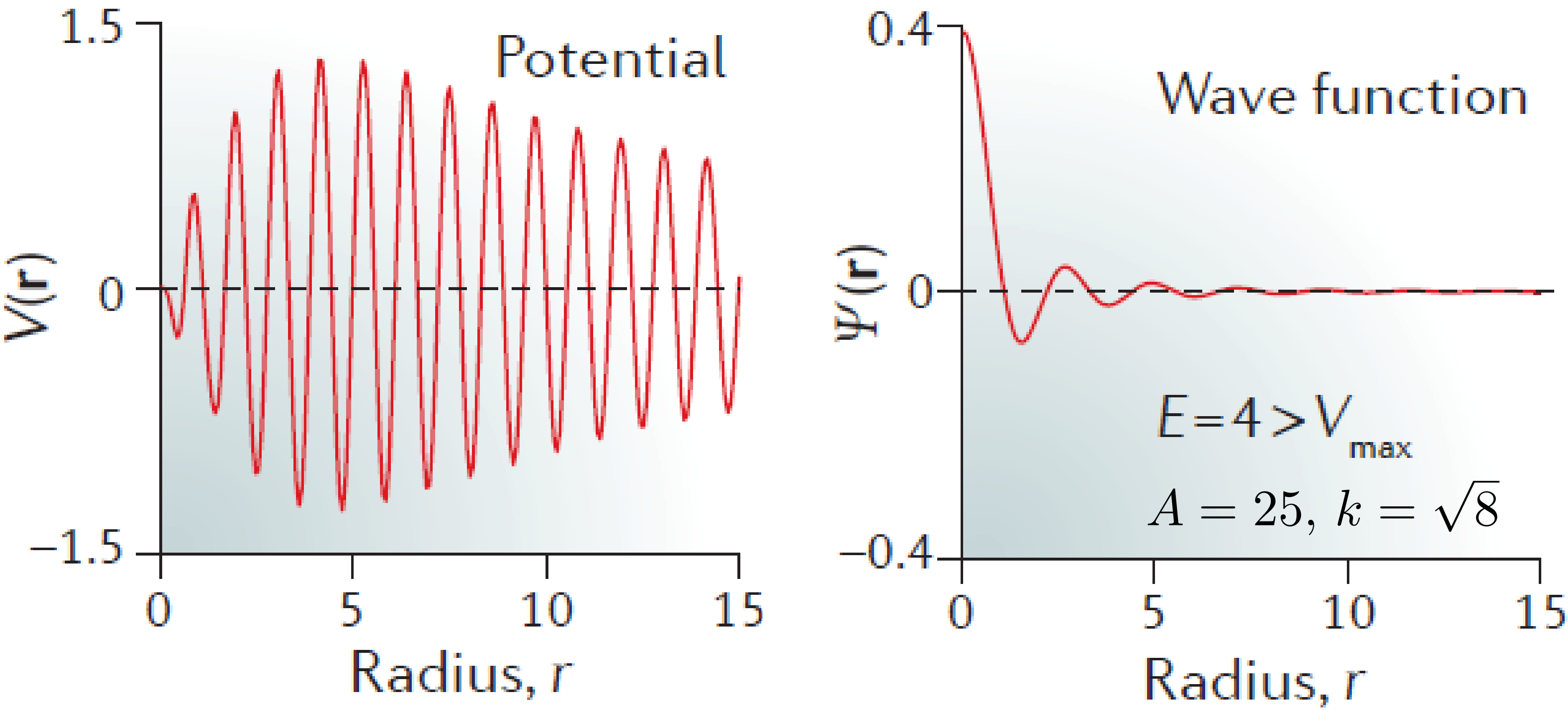
$$E = \frac{k^2}{2} > 0, \quad \text{hence in the continuum}$$

$$\text{where } f(r) = [A^2 + (2kr - \sin(2kr))^2]^{-1}$$

$$\text{Soln: } V(r) = -\frac{64k^2A^2\sin^4 kr}{(A^2 + (2kr - \sin 2kr)^2)^2} + \frac{48k^2\sin^4 kr - 8k^2(2kr - \sin 2kr)\sin 2kr}{A^2 + (2kr - \sin 2kr)^2}$$

J. von Neuman *et. al.*, *Physikalische Zeitschrift* 30 (1929) 467–470

The first BIC proposed: via potential engineering



J. von Neuman *et. al.*, *Physikalische Zeitschrift* 30 (1929) 467–470

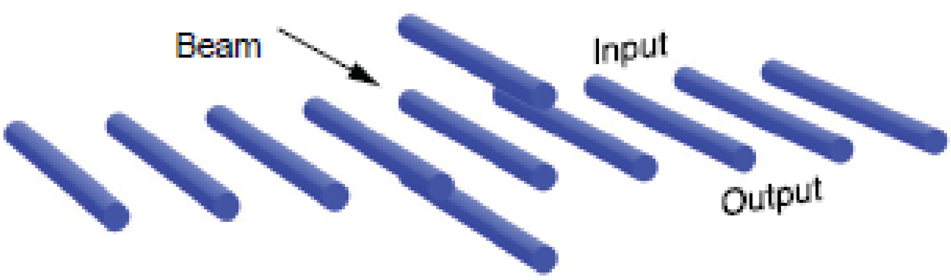
Via symmetry mismatch

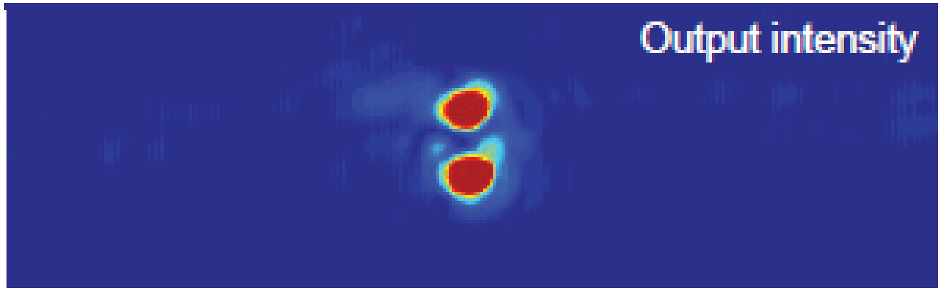
An 1D array of coupled optical waveguides: forms a continuum of modes.

Two waveguides placed above and below the array: Creates both a symmetric and an antisymmetric mode, embedded into the continuum.

The sym. mode can couple with the continuous energy band, hence leaks energy from waveguide pair to array.

The antisym. mode is decoupled from the continuous energy band, hence forming a bound state (energy cannot leak) embedded into a continuous spectrum of energy (formed by the array).





Fields bound inside the waveguide pair

Plotnik *et. al.*, PRL 107 (2011) 183901

Via separability

Consider $H = H_x(x) + H_y(y)$

We can consider the problem as 2 separate 1D Hamiltonian:

$$H_{x,y}\psi^n_{x,y} = E^n_{x,y}\psi^n_{x,y}$$

$$\psi_{tot} = \psi_x(x)\psi_y(y), \quad E_{tot} = E_x + E_y$$

Let H_x : infinite well, H_y: finite well of depth V_0; length a.

$$H_x \begin{cases} E_x = \dfrac{h^2 n^2 \pi^2}{2mb^2}\text{: all states bound} \\ \text{Ground state energy } E_g = \dfrac{h^2\pi^2}{2mb^2} \end{cases}$$

$$H_y \begin{cases} E_y < -V_0\text{: No solutions} \\ E_y \in [-V_0, 0]\text{: Bound states} \\ E_y > 0\text{: Continuum of states} \end{cases}$$

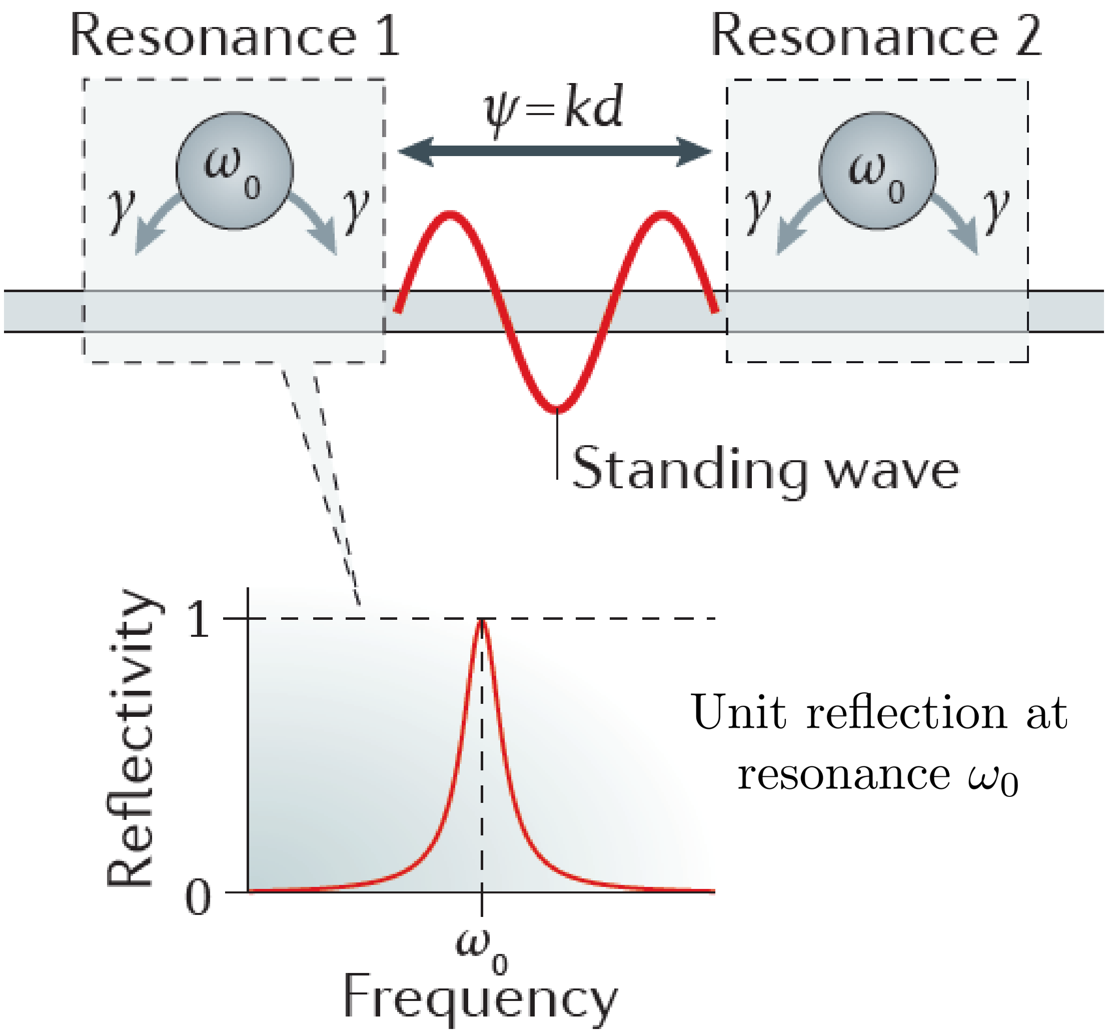
$\Rightarrow$ Continuum limit for tot. energy, $E_c = E_g + 0 = E_g$

Hence, a bound state in H_y with $E_y \in [-V_0, 0]$ and an excited state in H_x could have a total energy, $E_{tot} = E_x + E_y > E_c \Rightarrow$ inside the continuum.

Yet both $\psi_x(x)$ and $\psi_y(y)$ remains bound!

Robnik, Journal of Physics A: Mathematical and General 19 (1986) 3845-3848

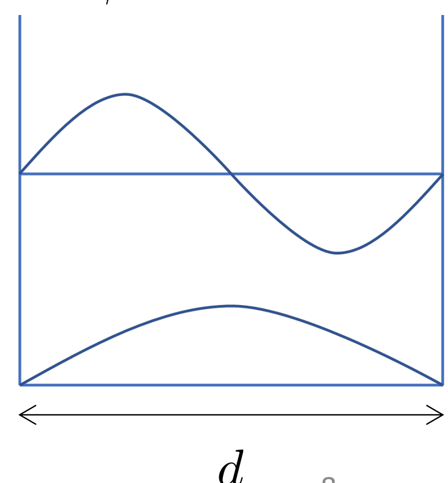
Fabry–Pérot BICs



Unit reflection at resonance ω_0

Trapped in between two mirrors

BIC formed when $\psi = kd = n\pi$



Hsu *et. al.*, Nature Rev. Mater 1 (2016) 16048

Math behind Fabry–Pérot BICs

resonance frequency of the uncoupled resonators

$$H = \begin{pmatrix} \omega_0 & \kappa \\ \kappa & \omega_0 \end{pmatrix} - i\gamma \begin{pmatrix} 1 & e^{i\psi} \\ e^{i\psi} & 1 \end{pmatrix}$$

linewidth/radiation rate
of individual resonances

coupling constant between resonators

$\psi = kd -$ propagation phase shift between two resonators

transverse wavenumber

$$\omega_\pm = \omega_0 \pm \kappa - i\gamma(1 \pm e^{i\psi})$$

Avoided crossing

When $\psi = n\pi$: No linewidth (BIC) and twice the linewidth

Friedrich–Wintgen BICs

When the two resonances are not separated, $d = 0$.

$$\omega_{\pm} = \omega_0 \pm \kappa - i\gamma(1 \pm 1)$$ ← BIC and leaky mode

$\Rightarrow$ Two resonances in the same structure can interfere to form a BIC!

For two general resonances, $$H = \begin{pmatrix} \omega_1 & \kappa \\ \kappa & \omega_2 \end{pmatrix} - i \begin{pmatrix} \gamma_1 & \sqrt{\gamma_1\gamma_2} \\ \sqrt{\gamma_1\gamma_2} & \gamma_2 \end{pmatrix}$$

$$\kappa(\gamma_1 - \gamma_2) = \sqrt{(\gamma_1\gamma_2)}(\omega_1 - \omega_2)$$

→ BIC and leaky mode

Friedrich and Wintgen.

Continuous evolution of parameters causes destructive interference between two resonances: avoided crossing.

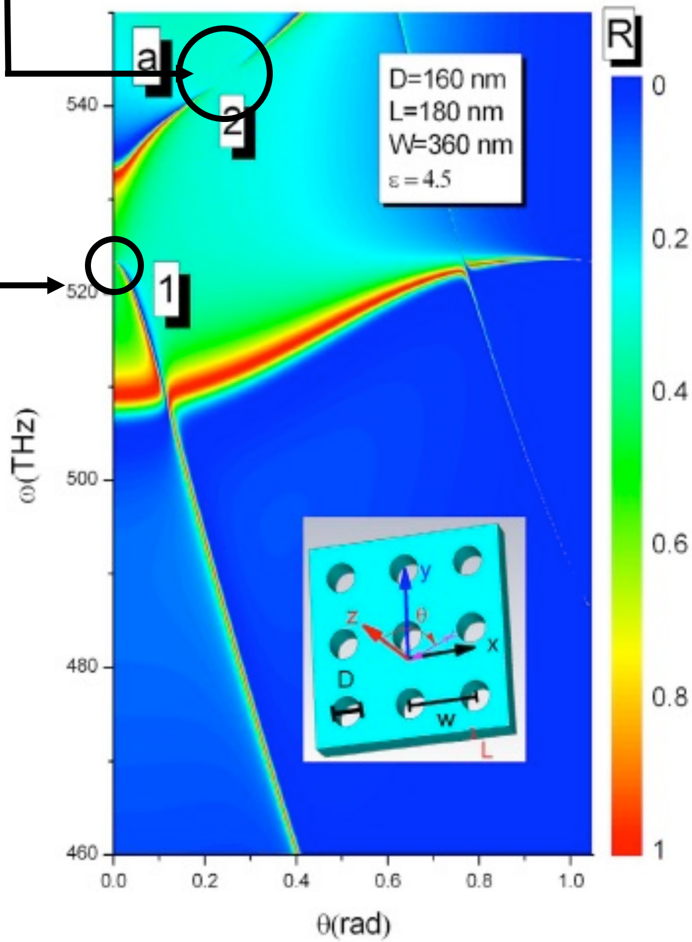
One of them turn into a BIC and the other into a leaky mode.

Friedrich et. al., Phys. Rev. A 32 (1985) 3231-3242

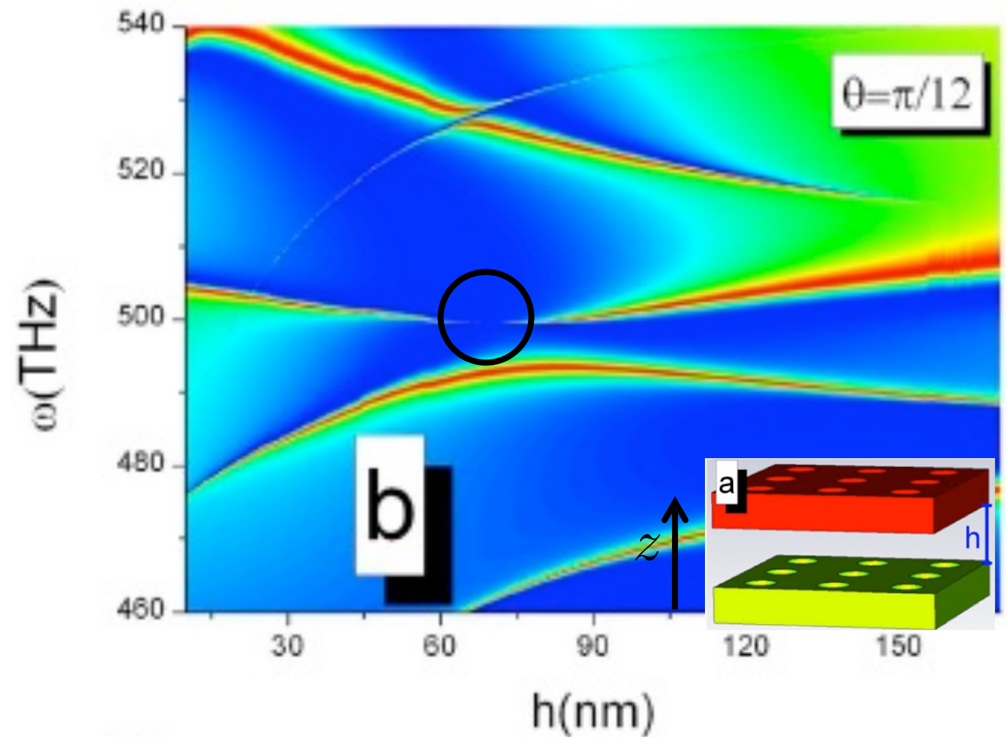
BIC in photonic crystals

A single PhC slab

BIC from continuous tuning of parameters.



Double layer PhC slab



BIC at $\theta = 0^0$. Due to symmetry mismatch between the mode and far-field radiation

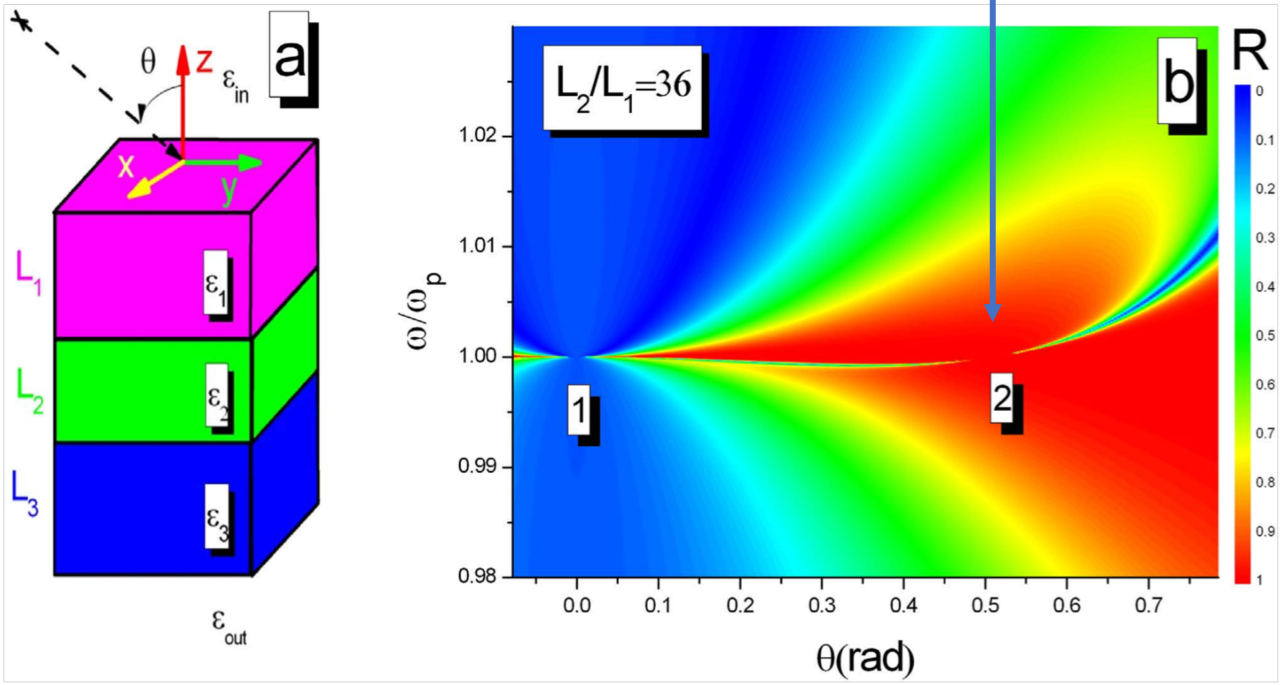
Li et. al., *Scientific Reports* (2016) 26988

BIC in metallic bilayers

Use L_2 dependence to control angle of BIC

BIC (Position controlled by L_2)

Metal



Drude Model: $\epsilon = 1 - \dfrac{\omega^2}{\omega_p^2} \Rightarrow$ Perfect reflector at ω_p

Li et. al., *Scientific Reports* (2016) 26988

Berreman modes

Numerically computed reflectance

Experimentally observed reflectance

p-polarized light

s-polarized light

ω_L

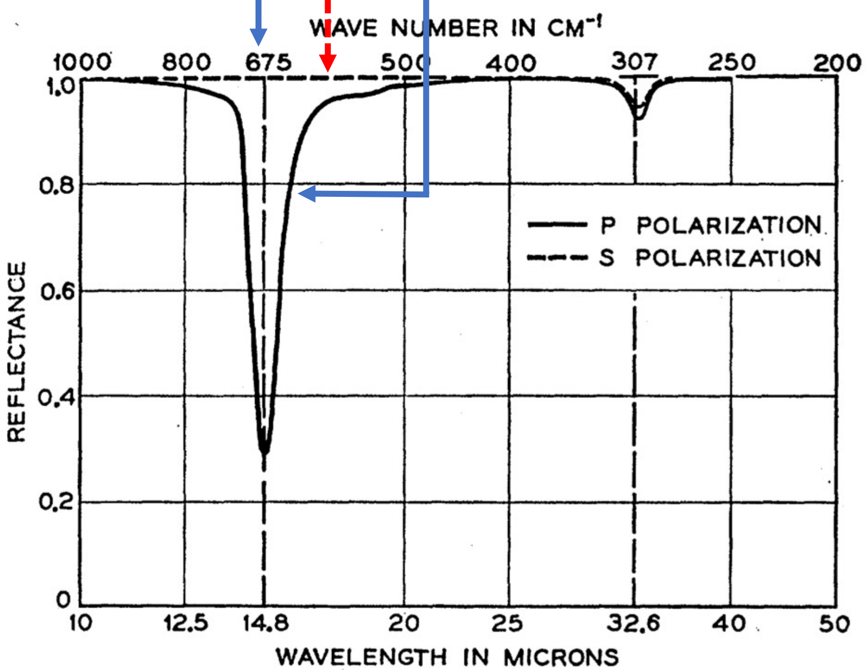
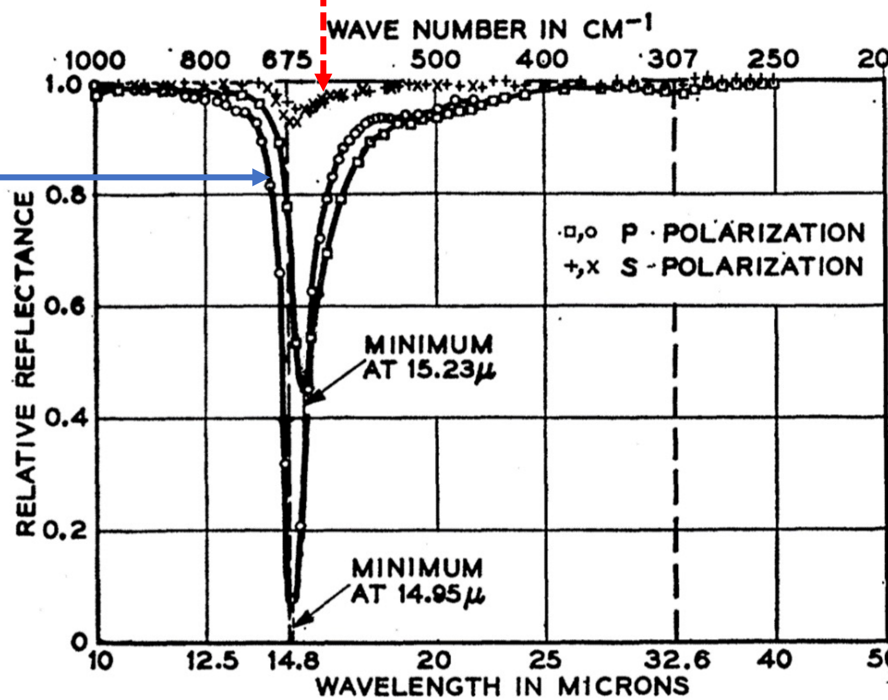


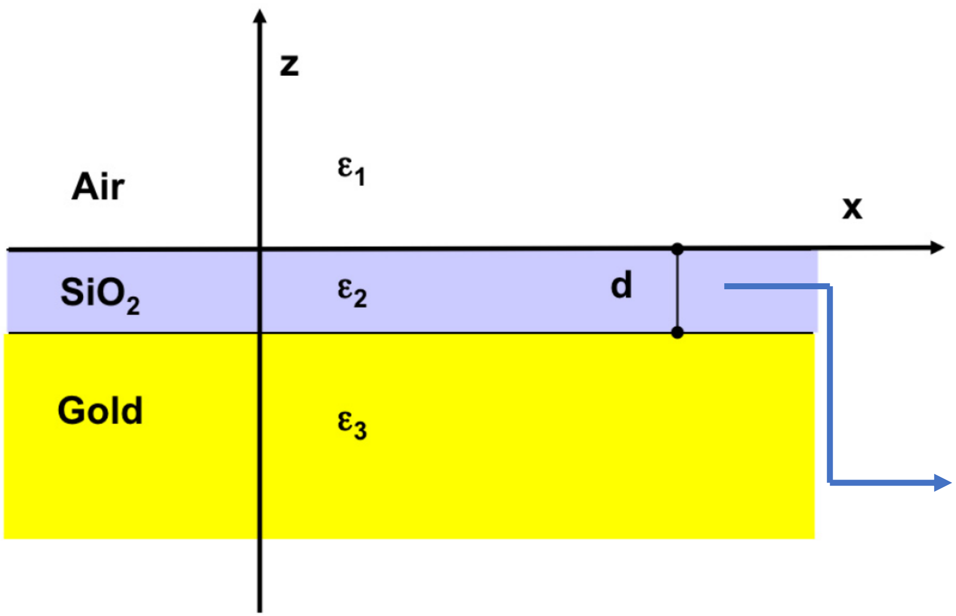
FIG. 2. Computed reflectance of *s*-polarized and *p*-polarized radiation by a LiF film 0.35 μ thick deposited upon silver; radiation incident at 30 deg.



LiF film on Ag substrate

Berreman, *Phys. Rev.* (1963) 2193

Berreman and ENZ modes



Solution to dispersion relation:

Complex ω + real $k_{||}$

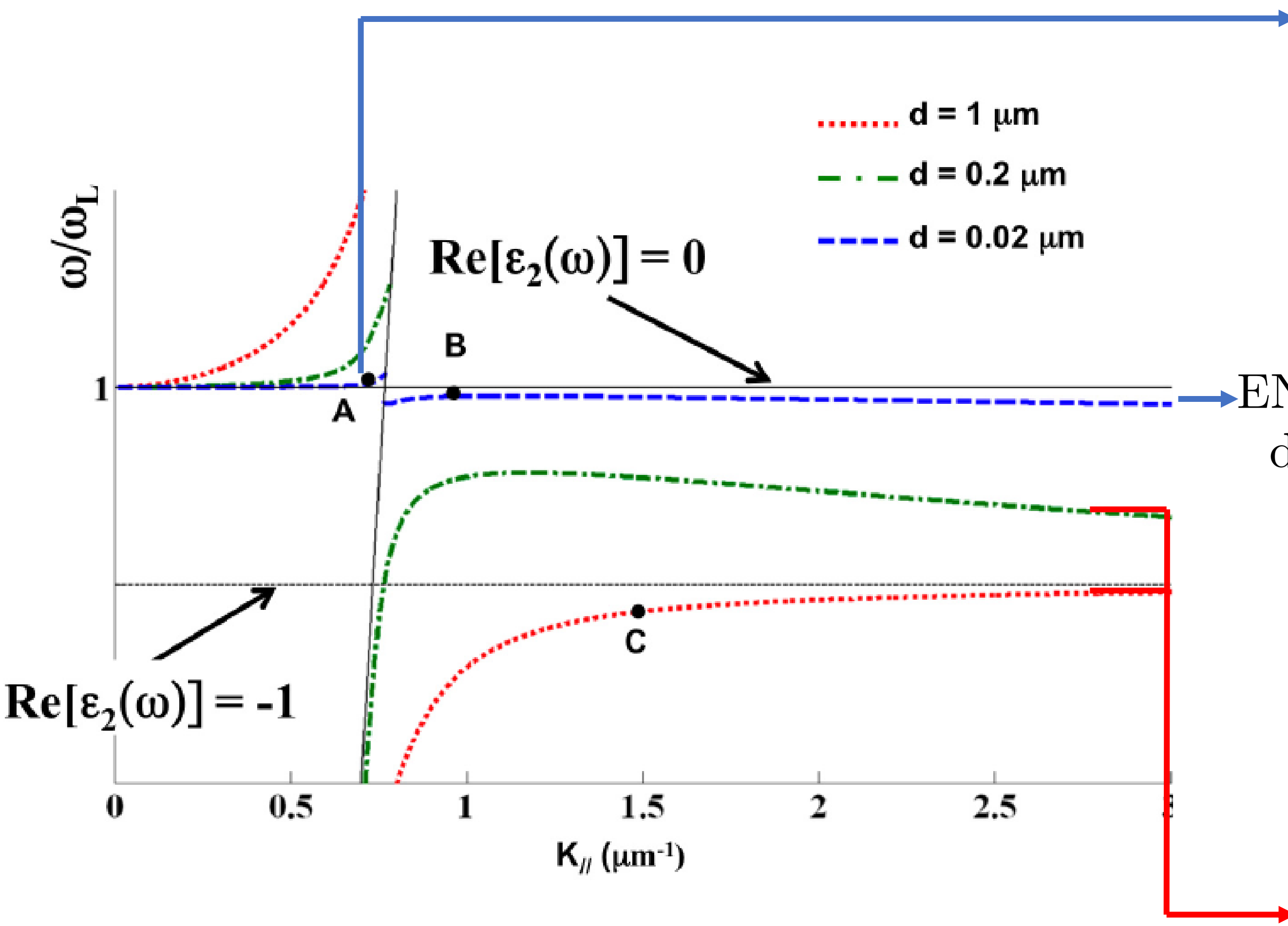
$$\epsilon_{SiO_2} = \epsilon_\infty \frac{\omega^2 - \omega_L^2 + i\omega\Gamma}{\omega^2 - \omega_T^2 + i\omega\Gamma}$$

Re $\epsilon_{SiO2} < 0$, for $\omega_L > \omega > \omega_T$

$\Rightarrow$ Air:SiO_2- surface phonon polariton modes

Vassant et. al., Opt. Express 20 (2012) 23971

Dispersion relation for SiO_2 on gold substrate



Berreman mode: Leaky

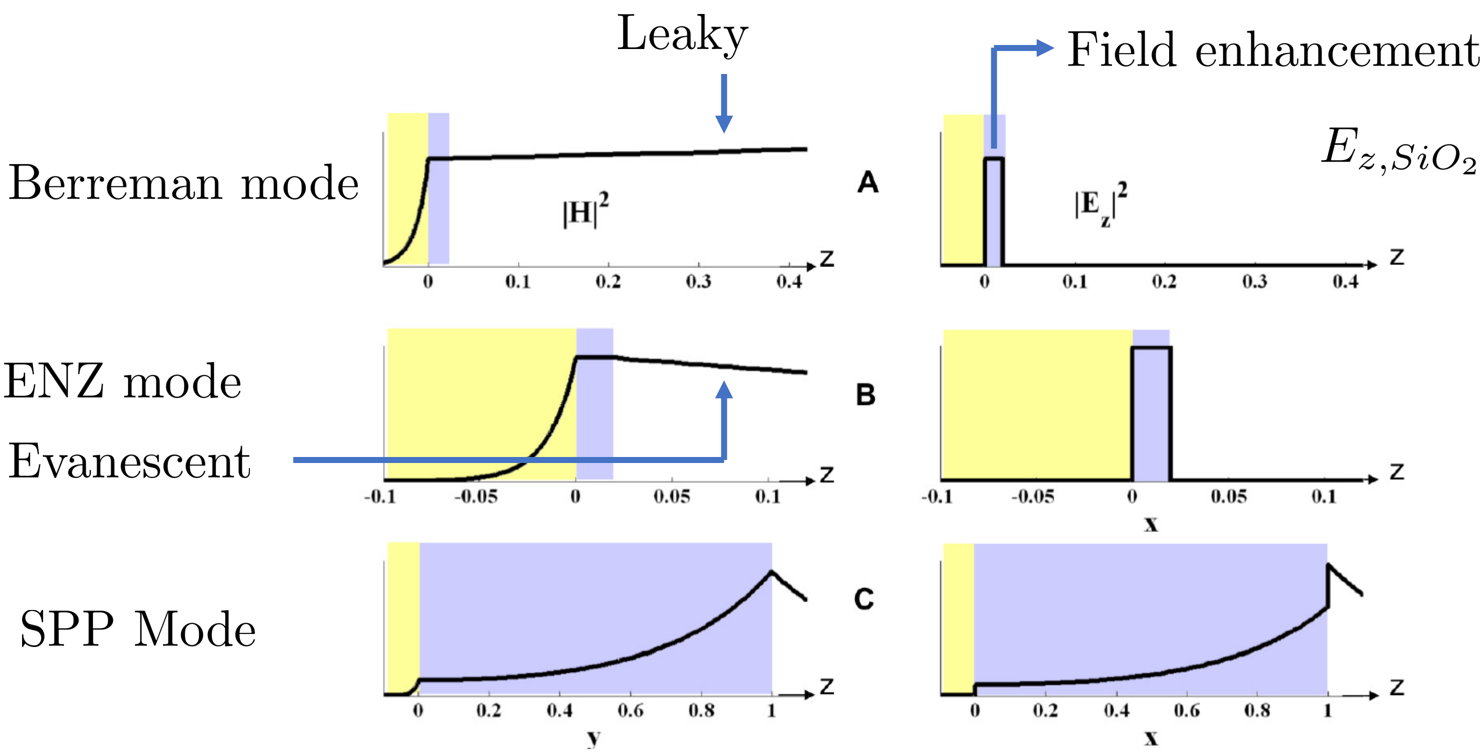
Coincides with Berreman absorption peak

ENZ Mode: SPP mode $\rightarrow \omega_L$ due to interaction between the two surfaces.

$k_{||} d_{SiO_2} \rightarrow$ large

$\Rightarrow$ tends to SPP mode at Air-SiO_2 interface

Berreman modes: Field enhancement



$$E_{z,SiO_2} = \frac{\epsilon_0}{\epsilon_{SiO_2}} E_{z,Air}$$

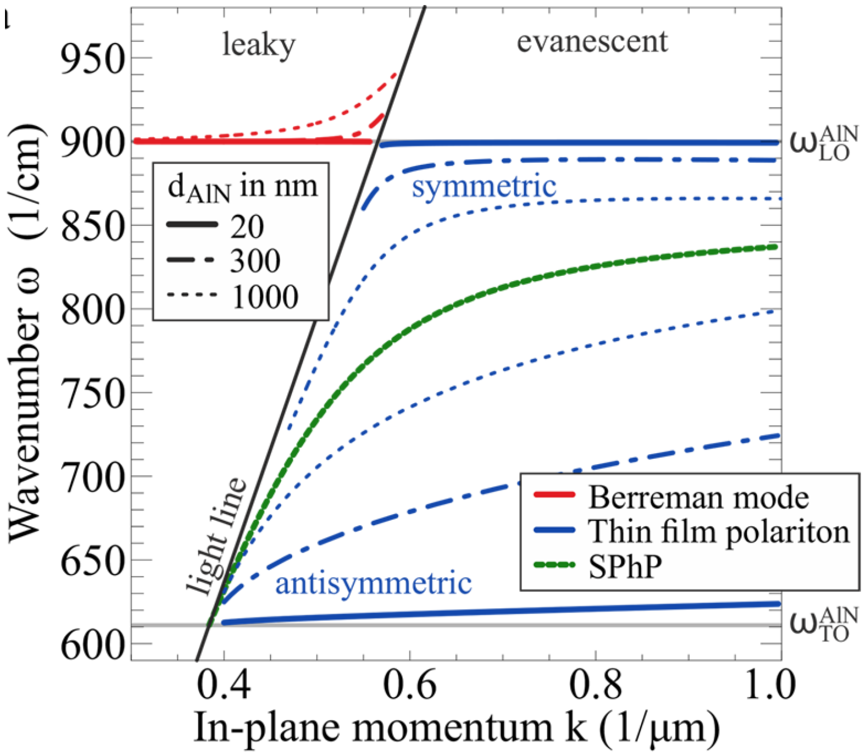
~ 0

Vassant et. al., Opt. Express 20 (2012) 23971

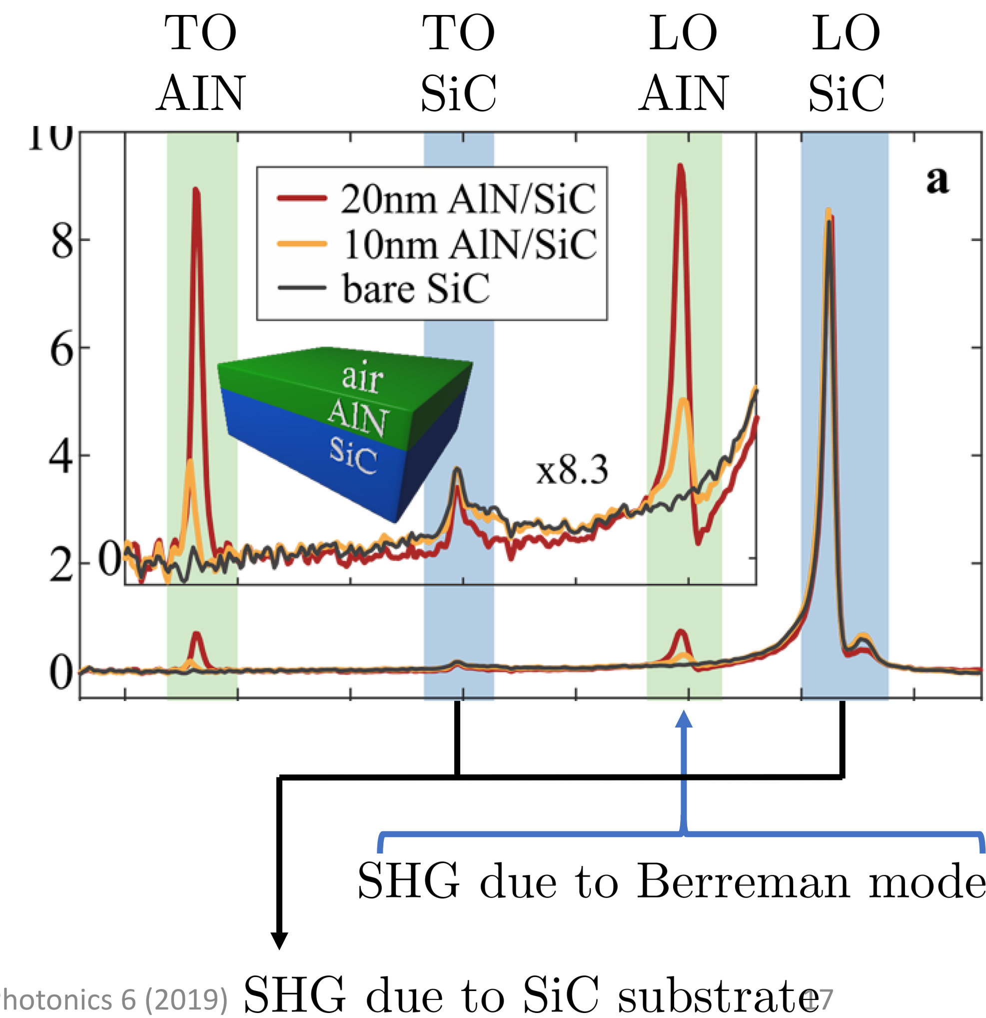
Berreman modes: Nonlinear applications

Field enhancements ⇒ nonlinear phenomena

Dispersion relation for AlN Film



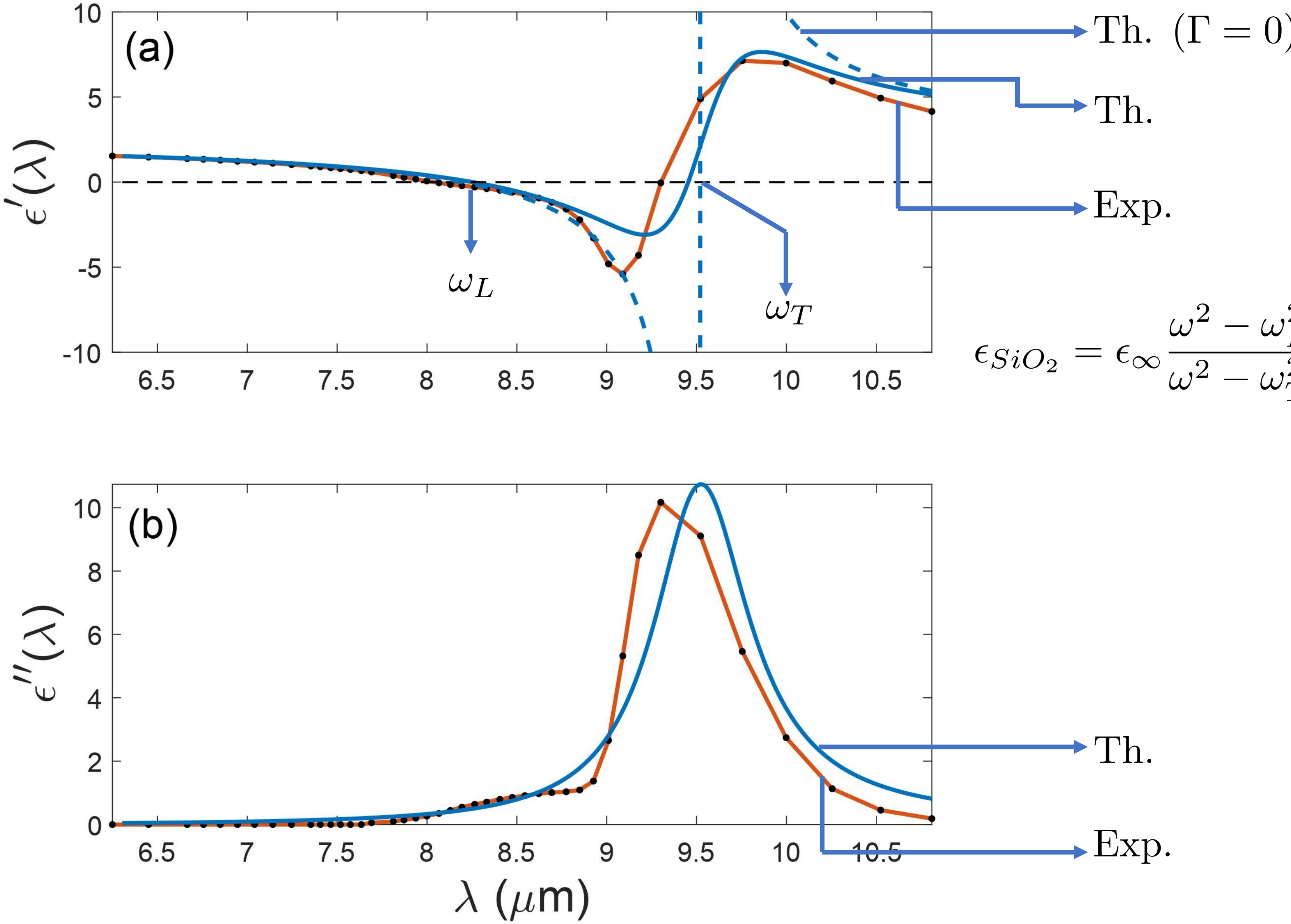
Experimental SHG in AlN film/ SIC substrate



SHG ⇒ experimental proof of field enhancement due to Berreman mode

Passler, ACS Photonics 6 (2019)

Permittivity of SiO_2



$$\epsilon_{SiO_2} = \epsilon_\infty \frac{\omega^2 - \omega_L^2 + i\omega\Gamma}{\omega^2 - \omega_T^2 + i\omega\Gamma}$$

SDG, Optics Communications 498 (2021) 127223

Dispersion relations

$$
\begin{aligned}
D_{sym}(k_x,\omega) &= m_{21} + m_{22}p_{tz} = 0 \\
D_{antisym}(k_x,\omega) &= m_{11} + m_{12}p_{tz} = 0
\end{aligned}
$$

$$p_{jz} = k_{jz}/k_0\epsilon_{rj} \quad k_{jz} = \pm\sqrt{\epsilon_{rj}k_0^2 - k_x^2}$$

$$k_x = k_0 \sin\theta$$

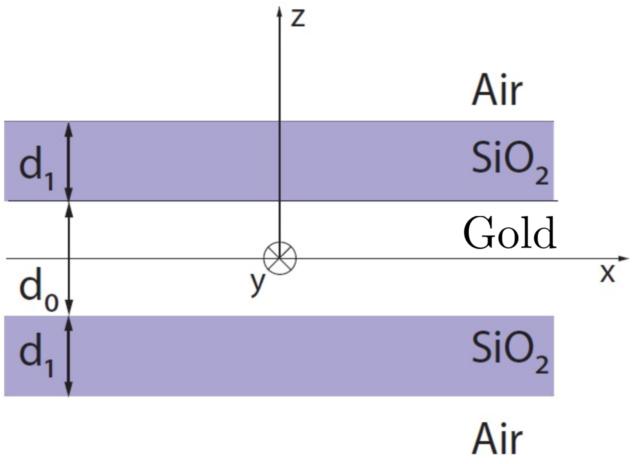
Angle of incidence

m_{ij}: elements of $M_T = M_0(d_0/2)M_1(d_1)$

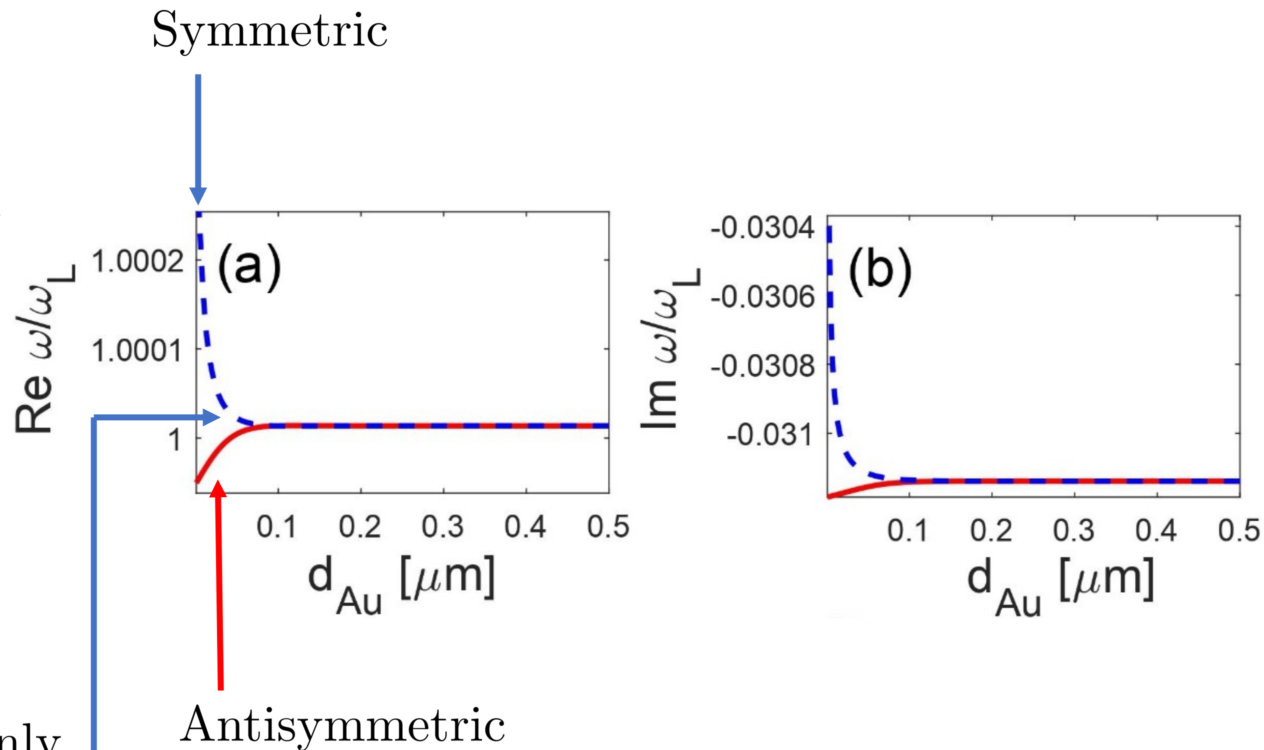
M_j: characteristic matrix of the jth layer.

$$M_j = \begin{pmatrix} \cos(k_{jz}d_j) & -(i/p_{jz})\sin(k_{jz}d_j) \\ -ip_{jz}\sin(k_{jz}d_j) & \cos(k_{jz}d_j) \end{pmatrix}.$$

Coupled Berreman modes: evanescent coupling



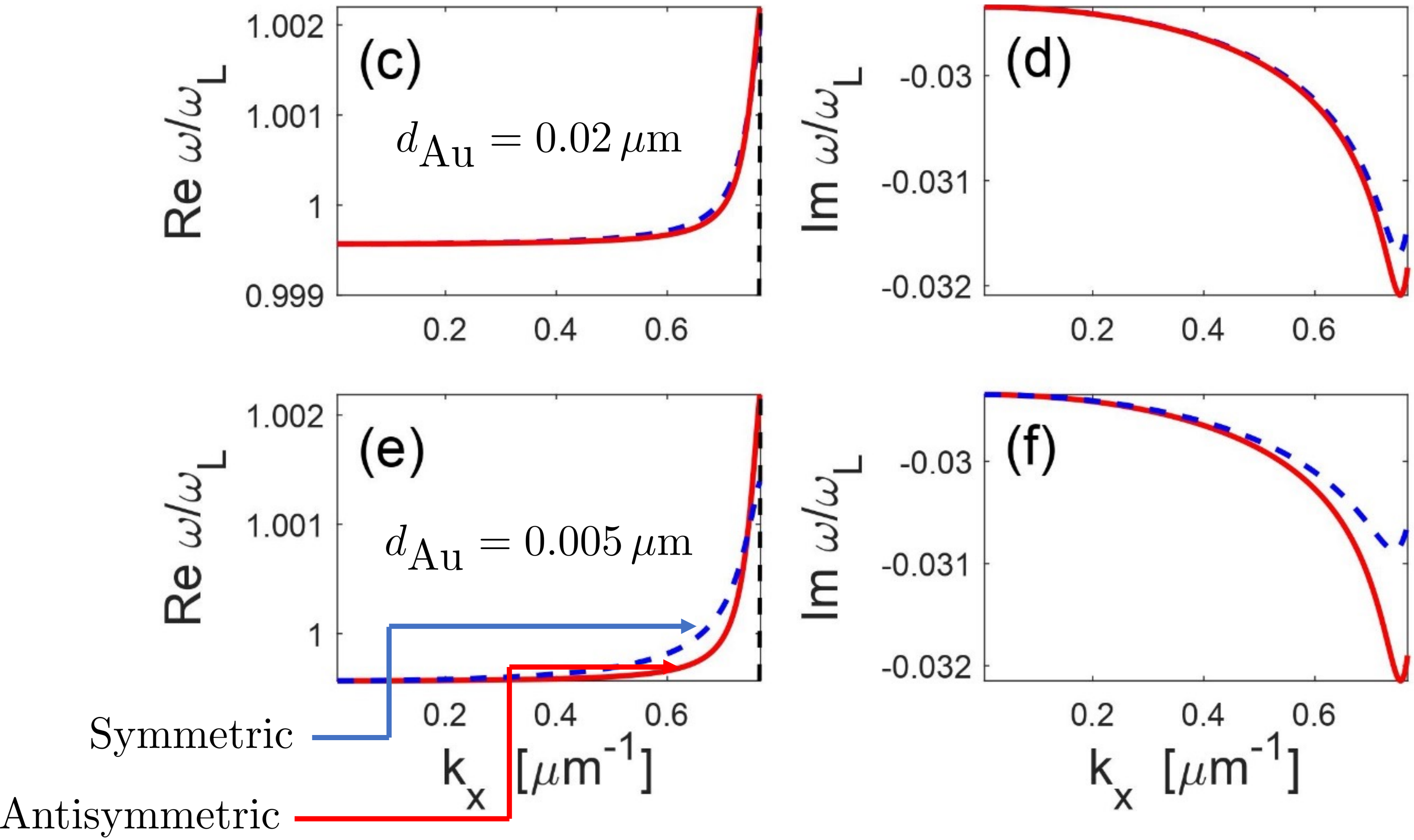
$\mathrm{Re}\,[\epsilon_{Au}] << 0 \Rightarrow$ Small skin depth



$\Rightarrow$ coupling possible for only low thickness Gold films

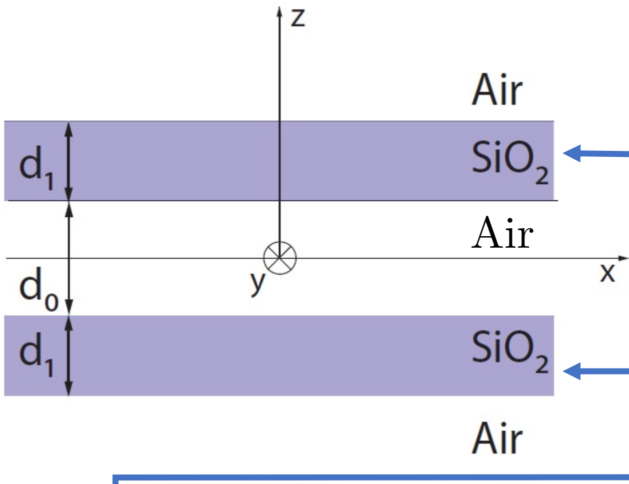
Normal mode splitting

Coupled Berreman modes: evanescent coupling (cond.)

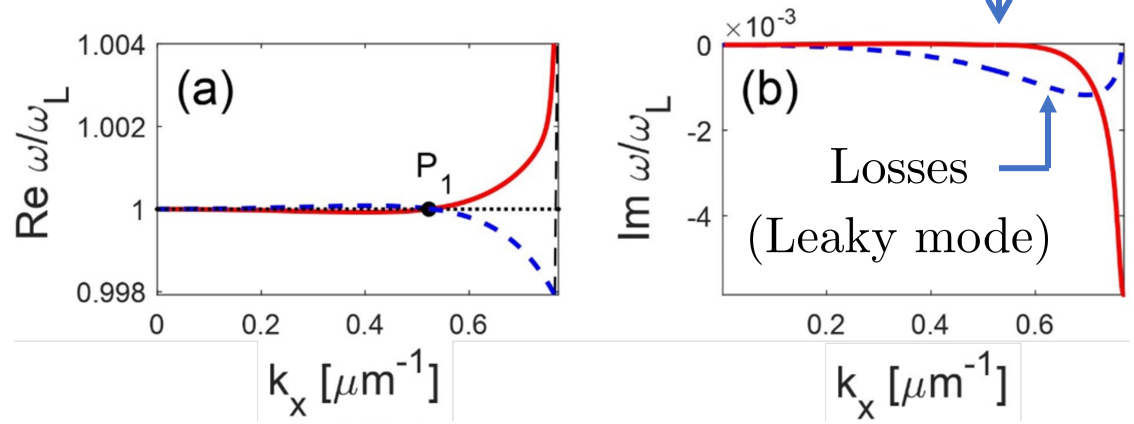


GR, PV, VGA, SDG, Optics Communications 498 (2021) 127223

Coupled Berreman modes: via propagation

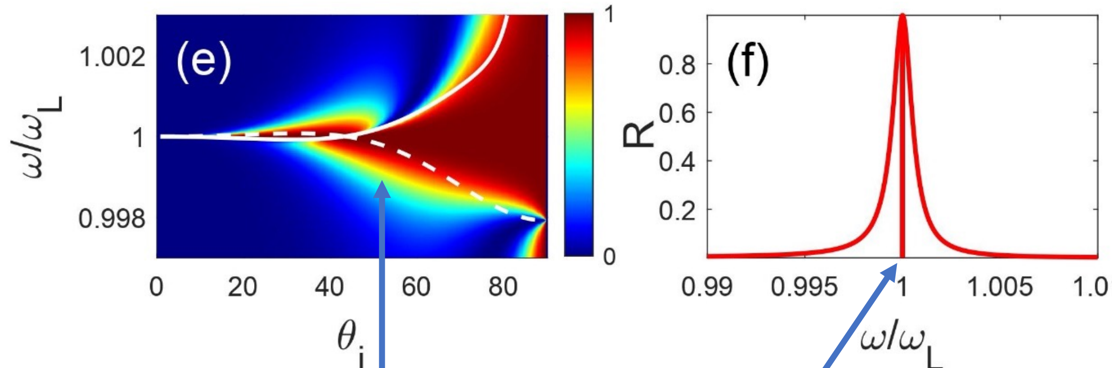


BIC



Loss-less $\Rightarrow$ Pair of perfect mirrors at ω_L (resonance)

$\Rightarrow$ Fabry–Pérot BICs

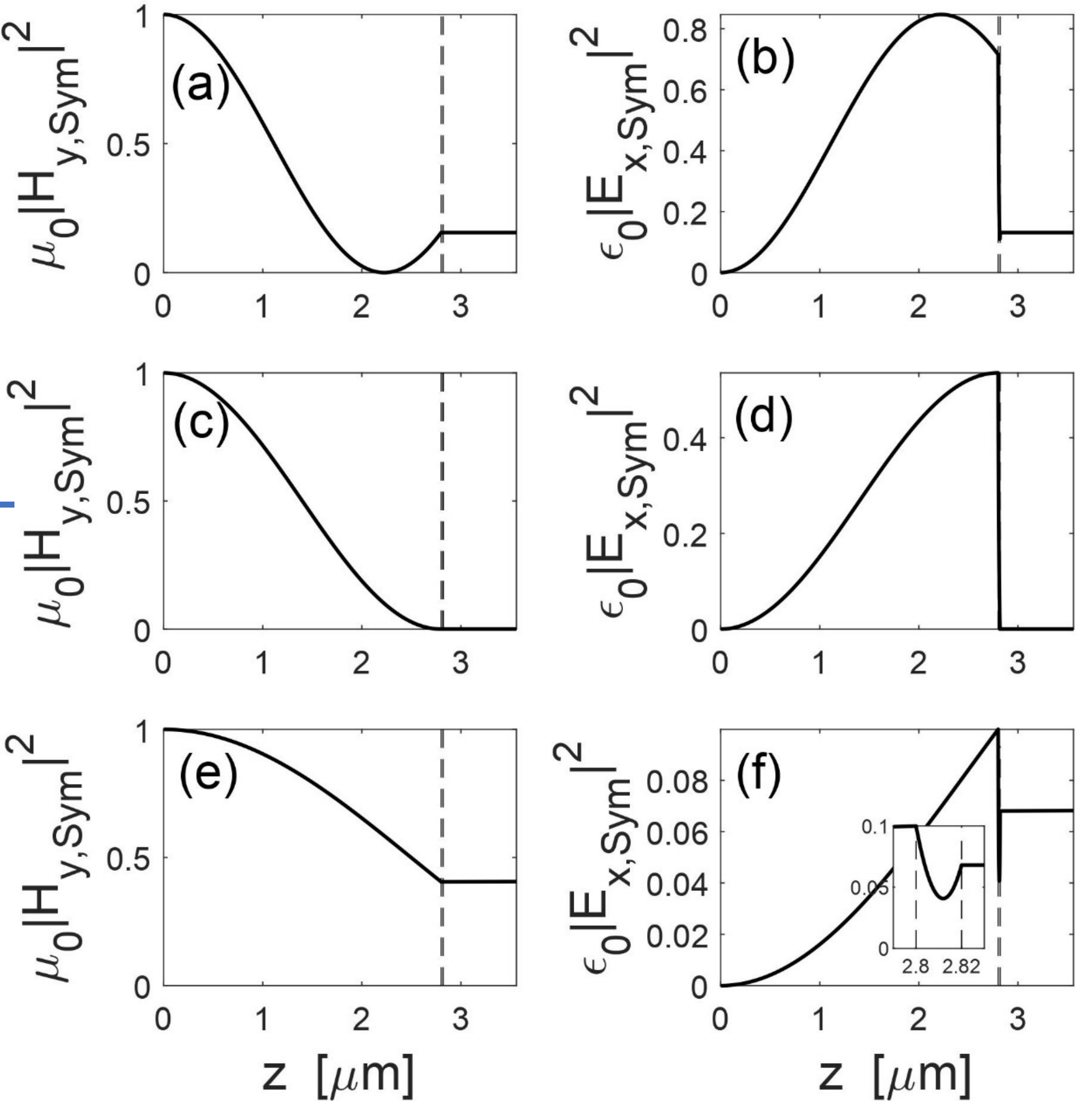


Null-width resonance

Fields at and near the BIC

Sym. mode away from BIC
(Before and after)

Sym. mode at point of
zero loss:
Completely confined⇒ BIC



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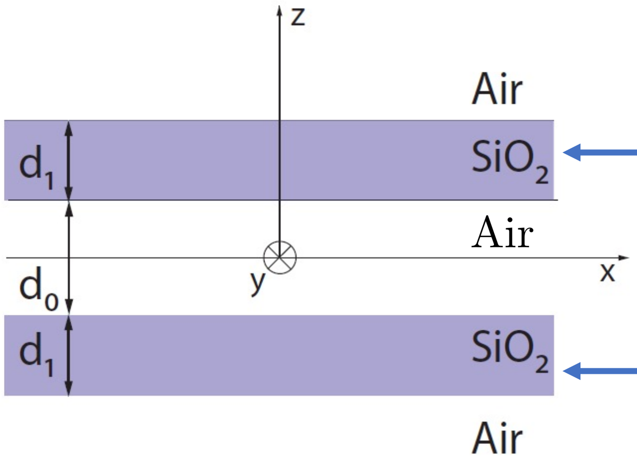
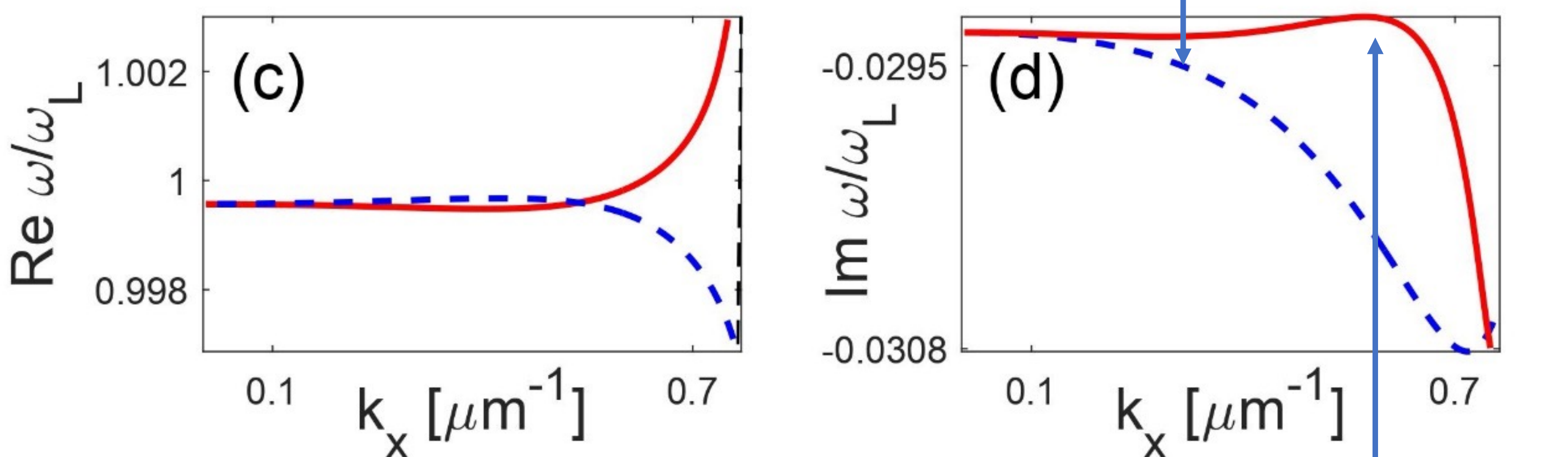
Realistic scenario



Lossy

Both modes lossy (radiative)

$\Rightarrow$ Absence of BIC or Quasi-BIC due to high losses



Note: Crossing in real parts $\Rightarrow$ Repulsion of imag. parts

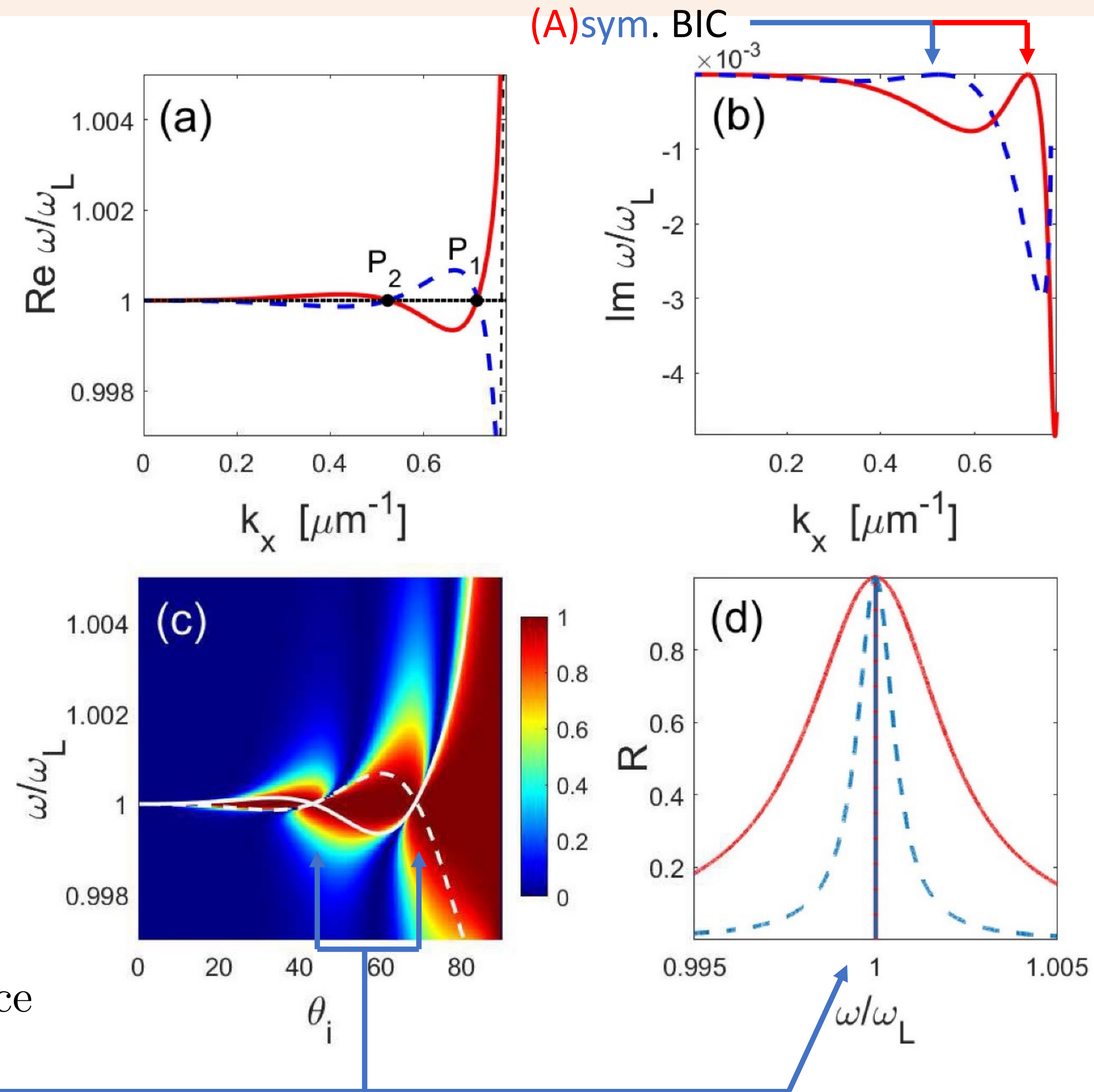
Highest Q factor $\sim$ 17

2^{nd} order BICs

Double the length of air layer

$\Rightarrow$ can fit two half-waves inside

$\Rightarrow$ Two BICs



Null-width resonance

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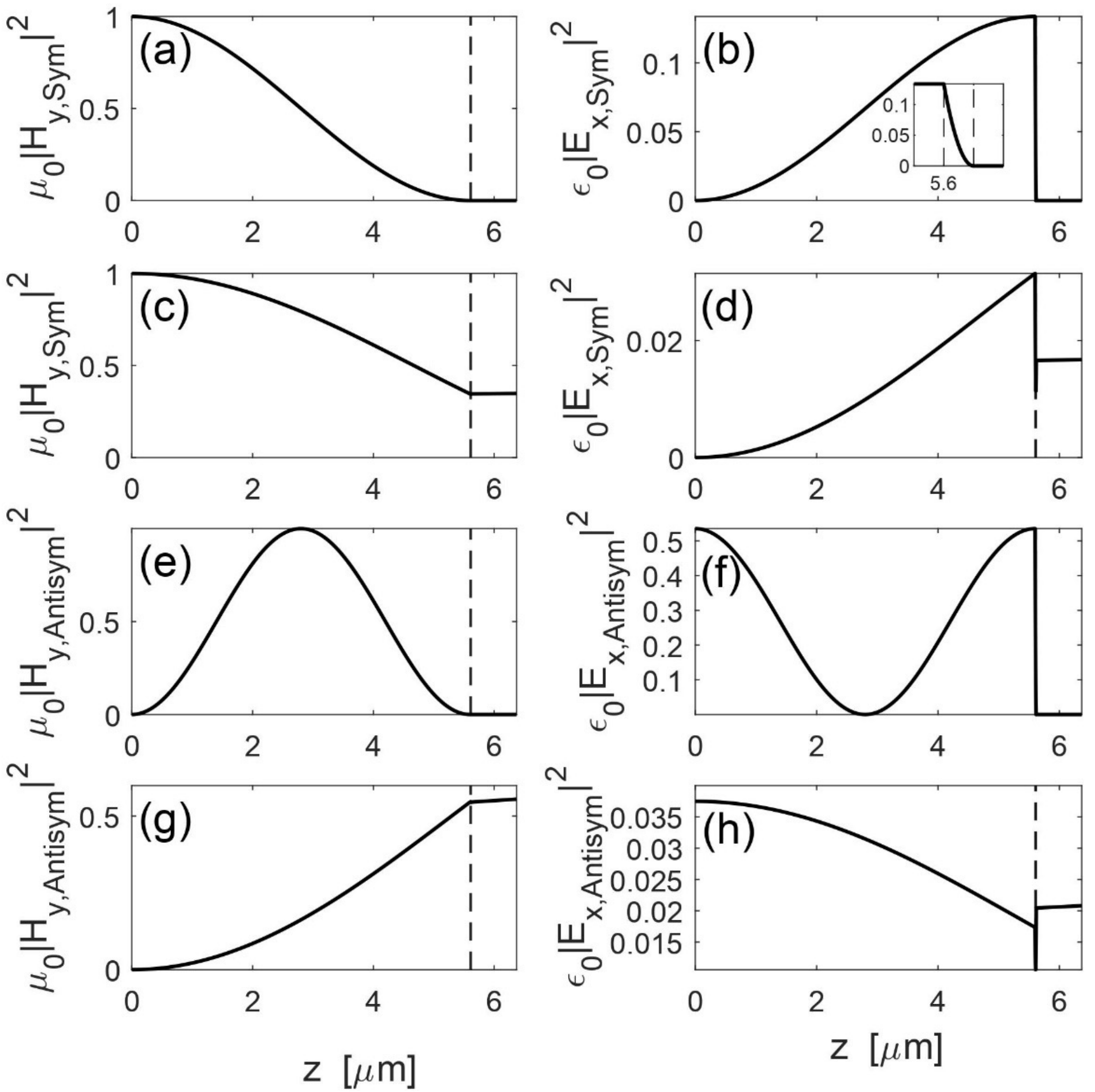
Fields at and near 1st and 2nd order BICs

Sym. mode at point of zero loss:

Completely confined $\Rightarrow$ 1^{st} order BIC

Antisym. mode at point of zero loss:

Completely confined $\Rightarrow$ 2^{nd} order BIC



Mathematical origin of higher order BICs

resonance frequency of the uncoupled resonators

$$H = \begin{pmatrix} \omega_0 & \kappa \\ \kappa & \omega_0 \end{pmatrix} - i\gamma \begin{pmatrix} 1 & e^{i\psi} \\ e^{i\psi} & 1 \end{pmatrix}$$

radiation rate of individual resonances

coupling constant

$$\omega_\pm = \omega_0 \pm \kappa - i\gamma(1 \pm e^{i\psi})$$

$$\psi = k_z d_0 = (2n+1)\pi$$

(1^{st}, 3^{rd}... order BICs (Sym.))

$$\omega_+ = \omega_0 + \kappa$$

$$\omega_- = \omega_0 - \kappa - 2i\gamma$$

$$\psi = k_z d_0 = 2n\pi$$

(2^{nd}, 4^{th}... order BICs (Antisym.))

$$\omega_+ = \omega_0 + \kappa - 2i\gamma$$

$$\omega_- = \omega_0 - \kappa$$

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Analytical solution for the dispersion relation

At ENZ: $\epsilon_{SiO_2} = 0$

$\omega = \omega_L$

Let $k_x = k_0 \sin\theta$

Angle of incidence

Sym. mode: Dispersion relation

$$-i\frac{p_{0z}}{p_{1z}}\cos(k_{1z}d_1)\sin\left(\frac{k_z d_0}{2}\right) - i\sin(k_{1z}d_1)\cos\left(\frac{k_z d_0}{2}\right)$$
$$+\frac{p_{0z}}{p_{1z}}\cos(k_{1z}d_1)\cos\left(\frac{k_z d_0}{2}\right) - \frac{p_{0z}^2}{p_{1z}^2}\sin(k_{1z}d_1)\sin\left(\frac{k_z d_0}{2}\right) = 0$$

$$\lim \epsilon_{SiO2} \to 0,\ \frac{1}{p_{1z}} = \frac{\epsilon_{SiO2}}{i\sin\theta} \to 0$$

$$\Rightarrow k_z d_0 = k_0 d_0 \cos\theta = \pi$$

Similarly, for Antisym. mode: $k_z d_0 = k_0 d_0 \cos\theta = 2\pi$

Conclusions

- We have presented a brief survey of BICs in various different physical situations.
- We have summarized some of the important results on Berreman modes and its applications.
- We looked at the properties of coupled Berreman modes, via both evanescent and propagating waves.
- We show how the BIC emerges in the system from these leaky modes.

G. Remesh, P. Vaity, V. G. Achanta and S. Dutta Gupta, "Exploring the route from leaky Berreman modes to bound states in continuum," vol. 498, p. 127223