Numerical Methods I

Midterm Exam

April 19, 2019

Total: 100 Marks

1. A and B are measurements with associated uncertainties (errors) δA and δB, respectively. C is a derived quantity with associated uncertainty δC. Derive expressions for the uncertainty δC for the following cases:

   (a) Addition of an exact (constant) number $\beta : C = A + \beta$,

   (b) Addition (or subtraction) : $C = A \pm B$,

   (c) Multiplication and division : $C = A \times B$, $C = \frac{A}{B}$,

   (d) Power law : $C = A^n$ ($n \neq 0$; n can be fractional or negative),

   (e) Exponential relationship : $C = \beta \exp(\alpha A)$.

   (20 Marks)

2. Consider the sequence of numbers

$$x_i = a_0 + 0.1i, \qquad (i = 1, ..., N). \tag{1}$$

   (a) The average of these numbers can be evaluated using the expressions

$$a_1 = \frac{\sum_{i=1}^{N} x_i}{N}; \qquad a_2 = a_0 + \frac{\sum_{i=1}^{N}(x_i - a_0)}{N}. \tag{2}$$

   (b) The second moment is given by

$$b = \frac{\sum_{i=1}^{N} x_i^2}{N}. \tag{3}$$

   Estimate the roundoff error in each of the above expressions.

   (20 Marks)

3. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix}. \tag{4}$$

   (a) Perform a direct triangular decomposition of A using the Gaussian elimination algorithm.

   (b) Perform an LU decomposition of A.

   (20 Marks)

4. The upward velocity of a rocket is measured as a function of time as

time (s)	velocity (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Determine the value of the velocity at $t = 35$ seconds using Lagrange interpolation with

(a) the first two data points,

(b) the first four data points,

(c) all six data points.

(20 Marks)

5. Consider the function $f(x)$ evaluated at three equally spaced points $x_0, x_1 = x_0 + h$ and $x_2 = x_0 + 2h$.

(a) Derive the three-point endpoint formula for the numerical derivative:

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]. \tag{5}$$

(b) Derive Simpson's rule for the numerical integral:

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}\left[f(x_0) + 4f(x_0+h) + f(x_0+2h)\right]. \tag{6}$$

(20 Marks)