Numerical Methods I

Assignment IV

Due: May 1, 2019

1. Find the roots of the function

$$f(x) = (x-3)(x-7), \tag{1}$$

using:

(a) bisection,

(b) fixed point iteration,

(c) the Newton-Raphson method.

Choose reasonable starting guesses x_i with $|x_i - \omega_i| \geq 1$, where $\omega_i = 3, 7$ are the actual roots. How does the error converge in each of these cases?

(15 Marks)

2. Compute

$$\int_0^1 \frac{\sin(x)}{x} dx, \tag{2}$$

using:

(a) the trapezoidal rule, with $h = \frac{1}{4}$,

(b) a three-term Gaussian quadrature formula,

(c) Simpson's rule.

(15 Marks)

3. The evolution equation for the probability of a random walker on an $N \times N$ periodic square lattice is given by

$$\frac{d}{dt}P(x,y,t) = \frac{1}{4}\left[P(x+1,y,t) + P(x-1,y,t) + P(x,y+1,t) + P(x,y-1,t)\right] - P(x,y,t). \tag{3}$$

Derive an expression for $P(x,y,t)$ given that the random walker begins at the origin, i.e. $P(x,y,0) = \delta_{x,0}\delta_{y,0}$. What is the limiting form at large times?

(20 Marks)