Numerical Methods I

Assignment II

Due: March 13, 2019

1. Estimate the error in evaluating the following expressions, where the input value of x has a relative error ϵ:

   (a) $x(1-x)$ $\quad x = 0.01, 0.1, 0.51, 0.6, 0.9, 0.99$

   (b) $\sqrt{1+x^2}$ $\quad x = 1, 100, 10000$

   (c) $\sqrt{1+x^2} + 100 - x$ $\quad x = 1, 100, 10000$

   (10 Marks)

2. Compute the sum $(x + x + ... + x)$, where $x = 1/3$ using floating point arithmetic with three decimal digits. What is the calculated value of the sum if the number of terms are 4, $30, 50, 300, 400$, and 1000.

   (10 Marks)

3. Consider the sum

$$s_n = \sum_{i=1}^{n/2} \frac{1}{2i(2i-1)}. \tag{1}$$

   Assuming that n is even, estimate the bounds on roundoff error for this series and compare them with the actual error.

   (15 Marks)

4. The derivative of a function is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \tag{2}$$

   Write a program to find this limit for the following functions, by taking $h = 2^{-n}, (n = 1, 2, ...)$:

   (a) $f(x) = e^x$ $\quad x = -1, 0, 1$

   (b) $f(x) = \sin(x)$ $\quad x = 0, \frac{\pi}{4}, \frac{\pi}{2}$

   (c) $f(x) = x^2 + 3x + 2$ $\quad x = 0, 1$

   (d) $f(x) = \frac{x^2+3x+2}{x+5}$ $\quad x = 0, 1$

   In each case, estimate the truncation and round-off error in the calculation and explain the results. Increase n until successive iterations yield a zero derivative (for $x \neq 0$). Use this value of n to estimate the number of bits stored in the fraction part of floating-point numbers on your computer.

   (15 Marks)