Condensed Matter Physics II

Assignment I

Due: March 9, 2020

1. **Partial Traces**
   Consider the tensor product of two systems. System 1 has two orthonormal basis states $|0\rangle$ and $|1\rangle$, whereas system 2 has three orthonormal basis states $|0\rangle$, $|1\rangle$, and $|2\rangle$. Consider the mixed state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |2\rangle + \frac{1}{\sqrt{6}}|1\rangle \otimes (|0\rangle + i|1\rangle + |2\rangle)\,. \tag{1}$$

   (a) Express $\rho_{12} = |\psi\rangle\langle\psi|$ as a matrix in a suitable orthonormal basis of the tensor product space.

   (b) Compute the reduced density operators (partial traces) of the first system (qubit) and the second system (qutrit).

   (c) Compute the eigenvalues of the reduced density operators.

   (20 Marks)

2. **Fermi Liquid Theory**
   Various currents in Fermi liquid theory are given by the following expressions (spin-independent):

$$\begin{aligned}
\mathbf{j} &= \frac{1}{V}\sum_{\mathbf{p}}(\nabla_{\mathbf{p}}\epsilon^0_{\mathbf{p}})\delta\bar{n}_{\mathbf{p}} && \text{particle current,}\\
\pi_{ij} &= \frac{1}{V}\sum_{\mathbf{p}}\left(p_i\frac{\partial\epsilon^0_{\mathbf{p}}}{\partial p_j}\right)\delta\bar{n}_{\mathbf{p}} && \text{momentum current,}\\
\mathbf{q} &= \frac{1}{V}\sum_{\mathbf{p}}\epsilon^0_{\mathbf{p}}(\nabla_{\mathbf{p}}\epsilon^0_{\mathbf{p}})\delta\bar{n}_{\mathbf{p}} && \text{energy current (heat).}
\end{aligned} \tag{2}$$

   where $\epsilon^0_{\mathbf{p}}$ is energy in global equilibrium, and $\bar{n}_{\mathbf{k}}$ is the deviation of the distribution function from local equilibrium, and includes interactions between quasiparticles.

   (a) Calculate the particle current for a *single* excitation at momentum $\mathbf{p}$. Hint: in the absence of quasiparticle interactions this would have been just the group velocity $v_p \approx v_f\hat{\mathbf{p}}$ of the particle. With quasiparticle interactions it will be a different velocity $\mathbf{u}$, that besides $\mathbf{v}_{\mathbf{p}}$ includes backflow currents from all other quasiparticles disturbed by the motion of the original one.

   (b) Calculate the momentum and energy currents. For momentum current, use symmetry arguments to identify Fermi liquid parameters $F_{s,a}$ that enter π_{ij}.

   The following expressions might help:

$$\int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i = 0 \quad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j = \frac{1}{3}\delta_{ij} \quad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j\hat{k}_m = 0 \quad \int\frac{d\Omega_{\hat{k}}}{4\pi}\hat{k}_i\hat{k}_j\hat{k}_m\hat{k}_n = \frac{1}{15}(\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{jm}\delta_{in}).$$

   (30 Marks)